Waves and Oscillations

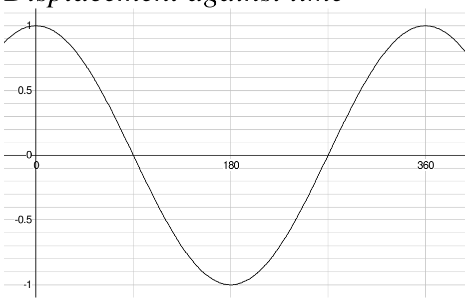
Circular Motion

- Angular speed ω is the angle though which something turns per second.
- $\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$ where θ is in radians
- since $T = \frac{1}{f}$ $\omega = 2\pi f$
- Where T is the period (the time taken for one revolution) and f is the frequency (the number of revolutions in one second).

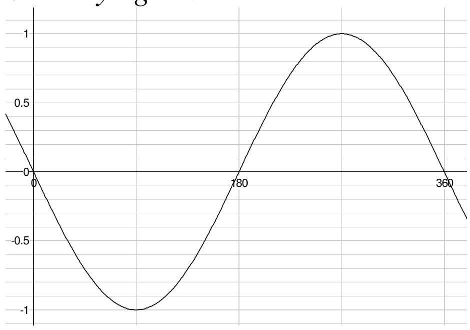
Simple Harmonic Motion

- SHM is motion in which the acceleration is directly proportional to the displacement from a fixed point and always directed towards that point.
- In SHM, period is independent of amplitude. Whether something's displacement is large or small its period is constant. This produces sinusoidal graphs, whereby all the graphs are of the same wavelength, but their amplitudes differ.

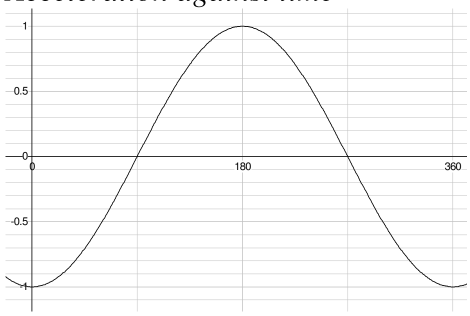
Displacement against time



Velocity against time



Acceleration against time



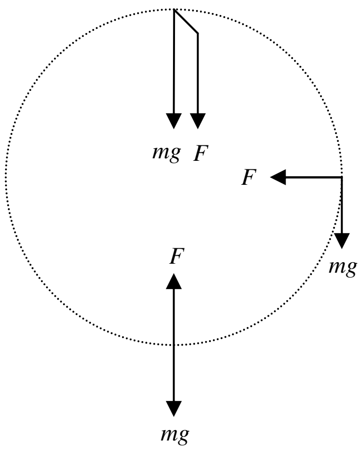
- These graphs show that when the displacement is 0, the acceleration is 0. Acceleration is always directed towards the equilibrium position.

$$a \propto -s$$
$$a = -sk$$
$$a = -\omega^2 x$$

- We can also see that displacement can be given by the formula $x = x_0 \cos\theta$ where x_0 is the maximum displacement. This can be proved via geometry when considering a circle. Since $\omega = \frac{\theta}{t}, \theta = \omega t$ and $\therefore x = x_0 \cos(\omega t)$
- With a simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$
- In a similar way the force exerted on a spring is given by the formula $F = -kx$. From this we can obtain $\omega^2 = \frac{k}{m}$ (where k is the spring constant). This being so, substituting into $\omega = \frac{2\pi}{T}$ we get $\sqrt{\frac{k}{m}} = \frac{2\pi}{T}$ or $T = 2\pi\sqrt{\frac{m}{k}}$ as the formula for the period of a mass suspended on a string.

Centripetal Force

- When moving in a circle there is an acceleration inwards known as the centripetal force.



As the item moves around the circle the resultant force differs. When at the top of the circle the only forces are acting downwards, which can result in a feeling of weightlessness.

- We can derive the formula $a = \frac{v^2}{r}$ for this acceleration, and from $F = ma$ we can deduce $F = \frac{mv^2}{r}$ as the formula for centripetal force.

Damping

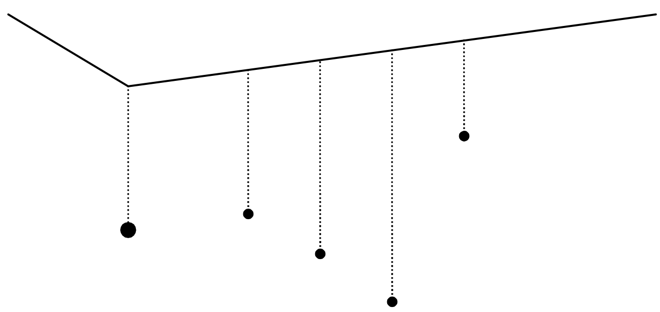
- In the situations previously considered we assume a frictionless environment, in which things would keep oscillating in perpetual motion indefinitely. This is just a modelling concept; in reality energy will always be lost from the system resulting in a gradual decrease of amplitude.
- With light damping oscillations gradually reduce in amplitude but take a long time to disappear completely, but with heavier or critical damping there is no oscillation, the object simply returns to its equilibrium position. With heavy damping this may take some time, but with critical damping it will happen as quickly as possible (using in approximately $T/4$ seconds).
- An application of this is shock absorbers.

Forced Oscillations and Resonance

- If the system is losing energy, it is possible to put this energy back by usage of a force. If the frequency of this applied force is the same as the natural frequency of the system then the amplitude of vibration increases.
- When the applied frequency matches the natural frequency resonance occurs. A large amount of energy is put into the system by the force, within mechanical structures this can be a problem.
- Resonance can be avoided by damping.

Coupled Pendula

- Barton's Pendula



- The pendulum with a length equal to that of the driver has the greatest amplitude, with all other pendula having lesser amplitudes as the length from the driver increases.
- Those pendula with longer strings were in phase, whilst those with the same length were in anti-phase, and those shorter were in phase.
- Phase difference is given by the formula $\Delta phase = \frac{2\pi\Delta path}{\lambda}$

Types of Wave

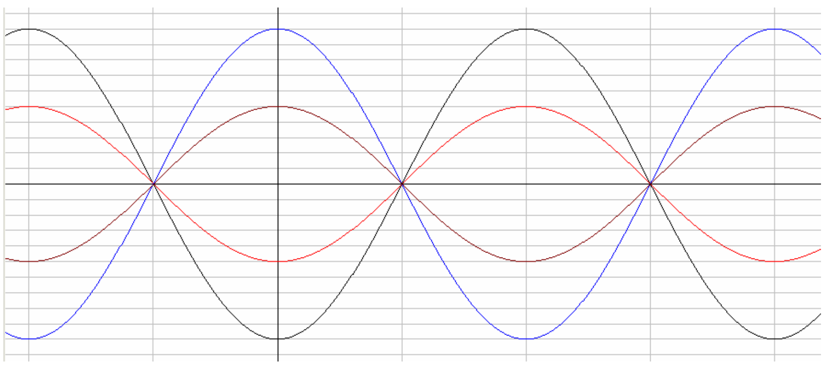
- Transverse waves
  - Displacement of particles is perpendicular to direction of wave.
  - Example being the EM spectrum where $c = f\lambda$
- Longitudinal waves
  - Displacement of particles is in the same direction as the wave
  - Examples are sound waves and water waves.
  - $v = f\lambda$

EM Waves

- The EM spectrum ranges from radio waves to gamma rays. It can be remembered with the phrase:
  - **R**andy : Radio
  - **M**ax's : Micro
  - **I**ntercourse : Infra Red
  - **L**eaves : Light
  - **U**tterly : UV
  - **E**xhuasted : X Rays
  - **G**irls : Gamma
- For EM waves the wave speed is the speed of light (obviously), which is 3.0×10^8 ms^{-1}

Standing & Progressive Waves

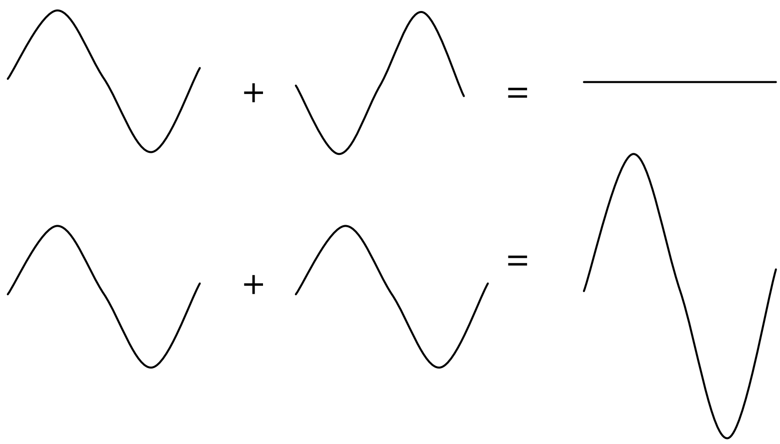
- Travelling waves transmit energy from one place to another, and whilst no particles actually *move* apart from displacement about an equilibrium point, there is the transfer of energy from one place to another.
- Standing waves are where the superposition of two waves generates points on the resultant waveform at which there is no displacement. This means there is no net transfer of energy.
- The graph below shows a standing wave system, there are points at which there is no displacement, even with varying amplitude. These points are known as nodes. The points at which the amplitude is greatest are anti-nodes.



- Nodes and anti-nodes are equidistance, and the distance between them should be equal to half the wavelength.
- In a standing wave all particles between nodes move in phase, however in a progressive wave they will move in anti phase.

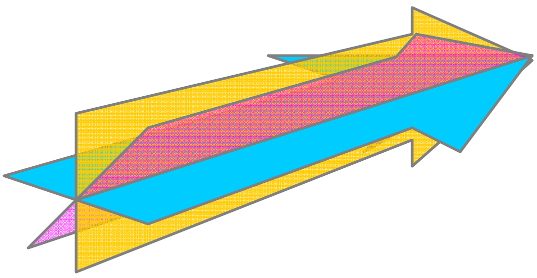
Phase and Superposition

- If things are in anti phase one wave is at its max when another is at its min.
- Adding waves which are in anti phase results in a net cancellation of the wave. Whereas adding waves with are in phase will result in a wave with a greater amplitude.

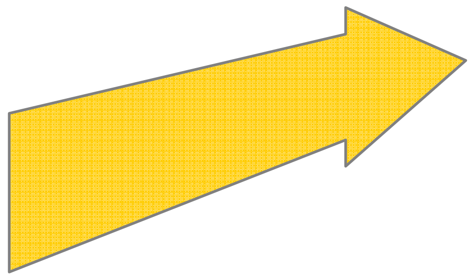


Polarisation

- In transverse waves particles move in a direction at right angles to the direction of the wave.
- However, these particles can be moving in different orientations – or planes – relative to the direction of the wave.
- When all the particles move in the same plane, a wave is said to be polarized.



This wave is not polarised, since particles are being displaced in three different planes.



This wave is polarised, since particles are being displaced in only one plane.

- Un-polarized waves can be polarized by a piece of polarized film ("Polaroid"). This film contains crystals which absorb waves in a given plane.
- White light is generally un-polarized, which waves travelling in random planes.
- When light is reflected by a transparent insulator such as glass the reflected (and refracted) light is partially polarized. When the reflected and refracted beams of light are at right angles to each other the reflected light is completely polarized. The angle of incidence which produces this effect is called Brewster's Angle.

Diffraction

- When light, like water, passes through a narrow slit it spreads out. When this spread out light is focuses onto a board, we can see that there are fringes in the light – indicating that the beams of light are interfering with each other.
- This is explained with the formula $\lambda = \frac{xs}{D}$, where x is the spacing between the maxima (point of maximum light intensity), s is the spacing between the slits and D is distance to the screen from which measurements are taken.

Wave Intensity

- The intensity of a wave from a point source is given by the formula $I = \frac{P}{d^2\pi}$