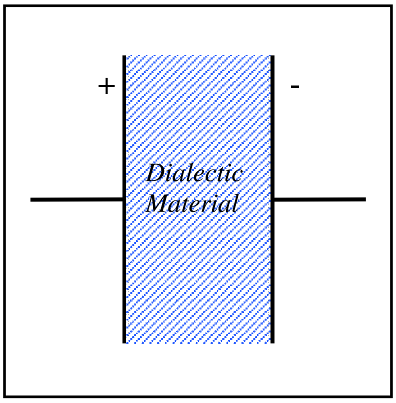
Capacitance



Capacitance is the measure of the size of a capacitor, defined by the surface area of the plates in the capacitor, and the plate separation. It also depend son the dielectric constant of the dielectric material which separates the two plates which make up a capacitor.

$$C = \frac{A\varepsilon_o}{d},\ Q = CV$$

To make the dielectric material more suitable (and thus the capacitor more efficient) the dielectric material should…

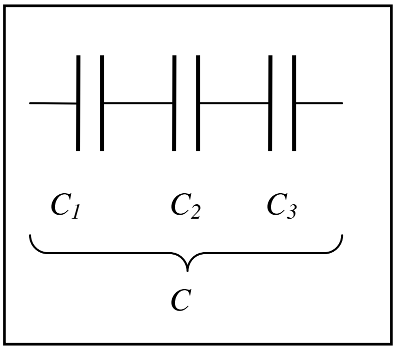
- Be an insulator
- Have polarised molecules

Energy from Capacitors

The electric potential at a point is the work done in moving a charge from 0 to ∞. The work done charging a capacitor is stored in the electric field of the capacitor.

$$\begin{aligned} \text{We know } Q &= CV \\ \text{and } \Delta U &= Q\Delta V \\ \text{so } U &= \tfrac{1}{2}QV \\ \text{using } Q &= CV \\ \text{we get } E = U &= \tfrac{1}{2}CV^2 \end{aligned}$$

Combining Capacitors in Series



On each capacitor; $Q_1 = Q_2 = Q_3 = Q$

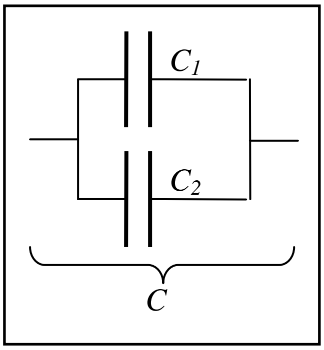
Kirchoff's Second; $V = V_1 + V_2 + V_3$

Using; $V = {}^{Q}\!/_{C}$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Combining Capacitors in Parallel



C

On each capacitor; $Q = Q_1 + Q_2$

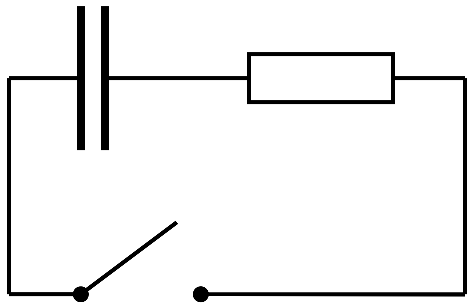
In Parallel; $V = V_1 = V_2$

Using; $V = {}^{Q}\!/_{C}$

$\therefore CV = C_1V + C_2V$

$C = C_1 + C_2$

Discharging a Capacitor through a Resistor



Initially,

Q_0 is the charge in the capacitor
V_0 is the voltage across the capacitor

At a time t during discharge,

V is the voltage across the capacitor
V_R is the voltage across the resistor
Q is the remaining charge

Since $\sum V = \sum emf$

$$0 = V_R + V$$

From $Q = CV$ and $V = IR$

$$0 = \frac{Q}{C} + IR$$

Since $I =$ Rate of change of charge

$$0 = \frac{Q}{C} + \frac{dQ}{dt}R$$

$$-\frac{Q}{C} = \frac{dQ}{dt}R$$

$$-\frac{Q}{RC} = \frac{dQ}{dt}$$

$$-\frac{dt}{RC} = \frac{dQ}{Q}$$

In the period $0 \le time \le t$

$$\frac{-1}{CR}\int_0^t dt = \int_{Q_0}^{Q} \frac{dQ}{Q}$$

$$\frac{-1}{CR}[t]_0^t = [\log_e Q]_{Q_0}^{Q}$$

Since $\log_e Q = \ln Q$

$$\frac{-1}{CR}(t) - \frac{-1}{CR}(0) = [\ln Q]_{Q_0}^{Q}$$

$$\frac{-t}{CR} = \ln Q - \ln Q_0$$

$$\ln(Q/Q_o) = \frac{-t}{CR}$$

$$\frac{Q}{Q_0} = e^{\frac{-t}{RC}}$$

$$Q = Q_0 e^{\frac{-t}{RC}}$$

Since $Q = CV$

$$Q_0 = CV_0$$

Substituting $\therefore$

$$CV = CV_0 e^{\frac{-t}{RC}}$$

$$V = V_0 e^{\frac{-t}{RC}}$$

RC is known as the "time constant" of the circuit.