

Edexcel Advanced Subsidiary GCE in

Mathematics (8371)

Further Mathematics (8372)

Pure Mathematics (8373)

For teaching from September 2004

First award in January 2005

Edexcel Advanced GCE in

Mathematics (9371)

Further Mathematics (9372)

Pure Mathematics (9373)

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First award in June 2005

Specification

June 2004

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Introduction

Key features

- ◆ All units equally weighted, allowing many different combinations of units and greater flexibility
- ◆ Options in mathematics and further mathematics
- ◆ Routes leading to full Advanced Subsidiary and Advanced GCE in Mathematics, Further Mathematics and Pure Mathematics
- ◆ Substantial INSET programme
- ◆ Mark schemes and examiners' reports available
- ◆ 18 units tested entirely by written examination
- ◆ CD ROM of past papers with an indication by each question of which unit it would lie in within the new Advanced GCE
- ◆ Endorsed textbooks and revision books, plus information available on how to map current textbooks to new Advanced GCE.

Candidates may study units leading to the following awards:

Advanced GCE in Mathematics	Advanced Subsidiary GCE in Mathematics
Advanced GCE in Further Mathematics	Advanced Subsidiary GCE in Further Mathematics
Advanced GCE in Pure Mathematics	Advanced Subsidiary GCE in Pure Mathematics
Advanced GCE in Further Mathematics (Additional)	Advanced Subsidiary GCE in Further Mathematics (Additional)

Unit availability

Examinations of each of the units will be available twice each year, in January and in June. Candidates may enter for any number of units at each examination session. Thus those candidates who wish to follow a traditional examination may take three units at the end of their Advanced Subsidiary course or six units at the end of their Advanced GCE course.

The units offered for examination will be staggered; units available in each examination session are shown below.

Examination Session	Units offered for examination
January 2005	C1, C2, FP1, M1, M2, M3, S1, S2, D1
June 2005	C1, C2, C3, C4, FP1, FP2, FP3, M1, M2, M3, M4, M5, S1, S2, S3, S4, D1, D2
January 2006 and all January sessions thereafter	C1, C2, C3, C4, FP1, M1, M2, M3, S1, S2, D1
June 2006 and all June sessions thereafter	All units

Rationale for the specification

In addition to the subject specific consultation meetings and questionnaires, a general consultation exercise was undertaken by QCA. The findings of this exercise, demonstrate that the specification meets the needs and expectations of centres in the ways identified below.

Flexibility in options and breadth in choice of topics.

As well as drawing on established knowledge, the subject matter of the specification is modern and contemporary.

The specification contains practical applications as well as theory, knowledge and understanding.

The course content and units of assessment are appropriate and accessible to all levels of ability.

There is continuity with current provision, which will allow use of existing resources.

A mixture of examination formats and question types is used, including short-answer questions and longer in-depth questions.

Summary of the scheme of assessment

There are 18 units, each designated as either Advanced Subsidiary or A2 units. The full Advanced GCE specification consists of 6 units, the Advanced Subsidiary specification of 3 units. These can be chosen from the 18 units available. Different combinations of units lead to different titles. It is not necessary for candidates taking an Advanced Subsidiary qualification to restrict themselves to units designated as Advanced Subsidiary.

All units consist of just one written paper.

Units

Title	Unit Code	Level	Method of assessment
Core Mathematics C1	6663	AS	1 written paper
Core Mathematics C2	6664	AS	1 written paper
Core Mathematics C3	6665	A2	1 written paper
Core Mathematics C4	6666	A2	1 written paper
Further Pure Mathematics FP1	6667	A2	1 written paper
Further Pure Mathematics FP2	6668	A2	1 written paper
Further Pure Mathematics FP3	6669	A2	1 written paper
Mechanics M1	6677	AS	1 written paper
Mechanics M2	6678	A2	1 written paper
Mechanics M3	6679	A2	1 written paper
Mechanics M4	6680	A2	1 written paper
Mechanics M5	6681	A2	1 written paper
Statistics S1	6683	AS	1 written paper
Statistics S2	6684	A2	1 written paper
Statistics S3	6670	A2	1 written paper
Statistics S4	6686	A2	1 written paper
Decision Mathematics D1	6689	AS	1 written paper
Decision Mathematics D2	6690	A2	1 written paper

All units equally weighted at $16\frac{2}{3}\%$

All examination papers of 1 hour 30 minutes

All examination papers out of 75 marks.

Summary of the specification content headings

Core Mathematics

C1	Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; differentiation; integration.
C2	Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; trigonometry; exponentials and logarithms; differentiation; integration.
C3	Algebra and functions; trigonometry; exponentials and logarithms; differentiation; numerical methods.
C4	Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; differentiation; integration; vectors.

Further Pure Mathematics

FP1	Inequalities; series; complex numbers; numerical solution of equations; first order differential equations; second order differential equations; polar coordinates.
FP2	Coordinate systems; hyperbolic functions; differentiation; integration.
FP3	Complex numbers; matrix algebra; vectors; Maclaurin and Taylor series; numerical methods; proof.

Mechanics

M1	Mathematical models in mechanics; vectors in mechanics; kinematics of a particle moving in a straight line; dynamics of a particle moving in a straight line or plane; statics of a particle; moments.
M2	Kinematics of a particle moving in a straight line or plane; centres of mass; work and energy; collisions; statics of rigid bodies.
M3	Further kinematics; elastic strings and springs; further dynamics; motion in a circle; statics of rigid bodies.
M4	Relative motion; elastic collisions in two dimensions; further motion of particles in one dimension; stability.
M5	Applications of vectors in mechanics; variable mass; moments of inertia of a rigid body; rotation of a rigid body about a fixed smooth axis.

Statistics

S1	Mathematical models in probability and statistics; representation and summary of data; probability; correlation and regression; discrete random variables; discrete distributions; the Normal distribution.
S2	The Binomial and Poisson distributions; continuous random variables; continuous distributions; samples; hypothesis tests.
S3	Combinations of random variables; sampling; estimation, confidence intervals and tests; goodness of fit and contingency tables; regression and correlation.
S4	Quality of tests and estimators; one-sample procedures; two-sample procedures

Decision Mathematics

D1	Algorithms; algorithms on graphs; the route inspection problem; critical path analysis; linear programming; matchings; flows in networks.
D2	Transportation problems; allocation (assignment) problems; the travelling salesman; game theory; dynamic programming.

Specification overview

The subject criteria

This specification incorporates the subject criteria for Mathematics as approved by QCA and which are mandatory for all awarding bodies.

Aims of the specification

The 18 units have been designed for schools and colleges to produce courses which will encourage candidates to:

- a develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment
- b develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs
- c extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems
- d develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected
- e recognise how a situation may be represented mathematically and understand the relationship between 'real-world' problems and standard and other mathematical models and how these can be refined and improved
- f use mathematics as an effective means of communication
- g read and comprehend mathematical arguments and articles concerning applications of mathematics
- h acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations
- i develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general
- j take increasing responsibility for their own learning and the evaluation of their own mathematical development.

Knowledge, understanding and skills

The knowledge, understanding and skills required for all Mathematics specifications are contained in the subject core. The units C1, C2, C3 and C4 comprise this core material.

Assessment objectives and weightings

	The assessment will test candidates' ability to:	Minimum weighting
AO1	recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.	30%
AO2	construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.	30%
AO3	recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.	10%
AO4	comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.	5%
AO5	use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.	5%

Key Skills

Whilst the examinations associated with this specification do not lend themselves to providing evidence for Key Skills assessment, candidates may be given the opportunity to meet many of the requirements through the employment of appropriate teaching and learning styles (see Appendix on page 80).

Progression and prior learning

Candidates embarking on Advanced Subsidiary and Advanced GCE study in Mathematics are expected to have covered all the material in the GCSE Mathematics Intermediate Tier. This material is regarded as assumed background knowledge and will not be tested by questions focused directly on it. However, it may be assessed within questions focused on other material from the relevant specification.

Synoptic assessment

Synoptic assessment in mathematics addresses candidates' understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, understanding and skills learned in different parts of the Advanced GCE course through using and applying methods developed at earlier stages of study in solving problems. Making and understanding connections in this way is intrinsic to learning mathematics.

In papers which address the A2 core content, synoptic assessment requires the use of methods from the AS core content. In papers which address mathematical content outside the core content, synoptic assessment requires the use of methods from the core content and/or methods from earlier stages of the same aspect of mathematics (pure mathematics, mechanics, statistics or discrete mathematics).

Synoptic assessment is addressed in the assessment objectives as parts of AO1, AO2, AO3 and AO4 (see page 7) and represents 20% of the assessment for Advanced GCE.

Synoptic material has been identified in the specification and is indicated with S for Advanced GCE Mathematics. All other material in these units has been indicated with N (non-synoptic). There is no requirement for synoptic material in Advanced GCE Further Mathematics.

Environmental and health education, the European dimension and spiritual, cultural and moral aspects

This specification will enable centres to provide courses in Mathematics that will allow candidates to discriminate between truth and falsehood. As candidates explore mathematical models of the real world there will be many naturally arising moral and cultural issues, environmental and safety considerations and aspects of European developments for discussion.

Forbidden combinations and related subjects

Any two subjects which have an overlap of at least one unit form a forbidden combination. Any two subjects with the same title also form a forbidden combination, whether or not there is any overlap of units.

Every specification is assigned to a national classification code indicating the subject area to which it belongs.

Centres should be aware that candidates who enter for more than one GCE qualification with the same classification code, will have only one grade (the highest) counted for the purpose of the School and College Performance Tables.

The classification codes for this specification are:

2210 Advanced Subsidiary GCE in Mathematics

2330 Advanced Subsidiary GCE in Further Mathematics

2230 Advanced Subsidiary GCE in Pure Mathematics

2210 Advanced GCE in Mathematics

2330 Advanced GCE in Further Mathematics

2230 Advanced GCE in Pure Mathematics

Qualification codes

Each qualification title, or suite of qualification titles with endorsements, is allocated two codes.

QCA codes

The QCA National Qualifications Framework (NQF) code is known as a Qualification Accreditation Number (QAN). This is the code that features in the DfES Funding Schedule – Section 96 and is to be used for all qualification funding purposes. Each unit within a qualification will also have a QCA NQF code.

The QCA qualification and unit codes will appear on the learner's final certification documentation.

The QAN numbers for qualifications in this publication are:

- 100/3142/2 Advanced GCE in Mathematics
- 100/3411/0 AS GCE in Mathematics

Edexcel codes

The Edexcel codes enable entry, assessment and certification, they will appear on documentation such as Centre Timetables, Statements of Entry, Attendance Registers, and Candidate Statements of Provisional Results. The Edexcel codes are not provided in this publication. The Edexcel codes will link automatically to the QCA codes for certification purposes.

Candidates with particular requirements

Regulations and guidance relating to candidates with particular requirements are published annually by the Joint Council for General Qualifications and are circulated to examinations officers. Further copies of guidance documentation may be obtained from the address below or by telephoning 0870 240 9800.

Edexcel will assess whether special consideration or concession can be made for candidates with particular requirements. Requests should be addressed to:

Special Requirements
Edexcel Foundation
Stewart House
32 Russell Square
London WC1B 5DN

Scheme of assessment

Advanced Subsidiary/Advanced GCE

All Advanced GCE Mathematics qualifications comprise six units and contain an Advanced Subsidiary subset of three units. The AS is the first half of an Advanced GCE course and contributes 50% of the total Advanced GCE marks. The A2, the second half of the Advanced GCE, comprises the other 50% of the total Advanced GCE marks.

Advanced Subsidiary awards are available for the titles Mathematics, Further Mathematics and Pure Mathematics.

Advanced GCE awards are available for the titles Mathematics, Further Mathematics and Pure Mathematics.

There are 18 units from which candidates may choose:

The following 7 units are the **Core** and **Further Pure Mathematics** units:

- C1 Core Mathematics C1 (AS)
- C2 Core Mathematics C2 (AS)
- C3 Core Mathematics C3 (A2)
- C4 Core Mathematics C4 (A2)
- FP1 Further Pure Mathematics FP1 (A2)
- FP2 Further Pure Mathematics FP2 (A2)
- FP3 Further Pure Mathematics FP3 (A2)

The 11 units below are the **Applications** units:

- M1 Mechanics M1 (AS)
- M2 Mechanics M2 (A2)
- M3 Mechanics M3 (A2)
- M4 Mechanics M4 (A2)
- M5 Mechanics M5 (A2)
- S1 Statistics S1 (AS)
- S2 Statistics S2 (A2)
- S3 Statistics S3 (A2)
- S4 Statistics S4 (A2)
- D1 Decision Mathematics D1 (AS)
- D2 Decision Mathematics D2 (A2)

All units except C1, C2, M1, S1 and D1 are designated as **A2**.

Title requirements

Conditions of dependency

The units in the areas of Pure Mathematics, Mechanics, Statistics and Decision Mathematics are consecutively numbered in order of study. The study of a unit is dependent on the study of all preceding units within that area of mathematics; for example, the study of C3 is dependent on the study of C1 and C2.

Candidates who wish to take Advanced Subsidiary or Advanced GCE in Further Mathematics may be expected to have obtained (or to be obtaining concurrently) an Advanced GCE in Mathematics. Units that contribute to an award in A level Mathematics may not also be used for an award in Further Mathematics. Candidates who have obtained or who are in the process of obtaining Advanced GCE in Mathematics with an awarding body other than Edexcel should contact the awarding body to check requirements for Further Mathematics.

Advanced subsidiary

Combinations leading to an award in Advanced Subsidiary level Mathematics comprise three AS units. Combinations leading to an award in Advanced Subsidiary level Further Mathematics comprise three units, including at least one A2 unit. The combination leading to an award in Advanced Subsidiary level Pure Mathematics comprises units C1, C2 and C3.

8371 Advanced Subsidiary Mathematics

Core Mathematics units C1 and C2 plus one of the Applications units M1, S1 or D1.

8372 Advanced Subsidiary Further Mathematics

Further Pure Mathematics unit FP1 plus two other units (excluding C1—C4). Candidates who are awarded certificates in both Advanced GCE Mathematics and AS Further Mathematics must use unit results from 9 different teaching modules.

8373 Advanced Subsidiary Pure Mathematics

Core Mathematics units C1, C2 and C3.

8374 Additional Qualification in Advanced Subsidiary Further Mathematics (Additional)

Candidates who complete 15 units will have achieved the equivalent of the standard of Advanced Subsidiary GCE Further Mathematics in their additional units. Such candidates will be eligible for the award of Advanced Subsidiary GCE Further Mathematics (Additional) in addition to the awards of Advanced GCE Mathematics and Advanced GCE Further Mathematics.

Advanced GCE

Combinations leading to an award in mathematics must comprise six units, including at least two A2 units. Combinations leading to an award in Further Mathematics must comprise six units, including at least three A2 units. Combinations leading to an award in Pure Mathematics comprise units C1—C4, FP1 and one of FP2 or FP3.

9371 Advanced GCE Mathematics

Core Mathematics units C1, C2, C3 and C4 plus two Applications units from the following six combinations: M1, M2; S1, S2; D1, D2; M1, S1; S1, D1; M1, D1.

9372 Advanced GCE Further Mathematics

Further Pure Mathematics units FP1, FP2, FP3 and a further three Applications units (excluding C1—C4) to make a total of six units; **or** FP1, either FP2 or FP3 and a further four Applications units (excluding C1—C4) to make a total of six units. Candidates who are awarded certificates in both Advanced GCE Mathematics and Advanced GCE Further Mathematics must use unit results from 12 different teaching modules.

9373 Advanced GCE Pure Mathematics

Core Mathematics units C1, C2, C3, C4, FP1, and either FP2 or FP3.

9374 Additional Qualification in Advanced Further Mathematics (Additional)

Candidates who complete 18 units will have achieved the equivalent of the standard of Advanced GCE Further Mathematics in their additional units. Such candidates will be eligible for the award of Advanced GCE Further Mathematics (Additional) in addition to the awards of Advanced GCE Mathematics, Advanced GCE Further Mathematics and Advanced Subsidiary GCE Further Mathematics (Additional).

Advanced Subsidiary

Title	Compulsory units	Optional units
Mathematics	C1, C2	One of M1, S1 or D1
Further Mathematics	FP1	Any two, excluding C1—C4
Pure Mathematics	C1, C2, C3	None

Advanced GCE

Title	Compulsory units	Optional units
Mathematics	C1, C2, C3, C4	One pair of units from: M1, M2 S1, S2 D1, D2 M1, S1 S1, D1 M1, D1
Further Mathematics	FP1 and one of FP2 or FP3	Four units (excluding C1—C4) to make a total of six units.
Pure Mathematics	C1, C2, C3, C4, FP1	Either FP2 or FP3

Number of awards

The maximum number of certificates for which a candidate is eligible is as follows:

3 units	one AS certificate
6 units	two certificates (one AS + one A OR two AS)
9 units	three certificates (two AS and one A)
12 units	four certificates (two AS and two A)
15 units	five certificates (three AS and two A)
18 units	six certificates (three AS and three A)

All 3, 6, 9, 12, 15 and 18 unit results must be used when claiming 1, 2, 3, 4, 5 and 6 certificates, respectively.

Calculators

Advanced Subsidiary and Advanced GCE awards in Mathematics and Pure Mathematics each include an element of the assessment, addressing assessment objectives AO1 and AO2, in which candidates are not permitted to use a calculating aid. This element must count for at least one unit of the overall award.

For unit C1, candidates may **not** have access to any calculating aids, including log tables and slide rules.

This specification is designed to encourage the appropriate use of graphic calculators and computers as tools by which the teaching and learning of mathematics may be enhanced.

Unit and resit rules

There is no restriction on the number of times a unit may be attempted prior to claiming certification for the qualification. The best available result for each unit will count towards the final grade.

Results of units will be held in Edexcel's unit bank for as many years as this specification remains available. Once the AS or Advanced level qualification has been certificated, all unit

results contributing to that qualification are deemed to be used up. These results cannot be used again towards a further award of the same qualification at the same level.

The full qualification at both Advanced Subsidiary and Advanced GCE may be resat more than once.

Awarding and reporting

The grading, awarding and certification of this specification will comply with the requirements of the current GCE Code of Practice for courses starting in September 2004, which is published by the Qualifications and Curriculum Authority. Qualifications will be graded and certificated on a five-grade scale from A to E. Individual results will be reported.

Language of assessment

Assessment of this specification will be available in English only. Assessment materials will be published in English only and all written and spoken work submitted for examination and moderation must be produced in English.

Relationship of assessment objectives to units

All figures in the following table are expressed as marks out of 75.

Assessment objective	AO1	AO2	AO3	AO4	AO5
Core Mathematics C1	30 – 35	25 – 30	5 – 15	5 – 10	1 – 5
Core Mathematics C2	25 – 30	25 – 30	5 – 10	5 – 10	5 – 10
Core Mathematics C3	25 – 30	25 – 30	5 – 10	5 – 10	5 – 10
Core Mathematics C4	25 – 30	25 – 30	5 – 10	5 – 10	5 – 10
Further Pure Mathematics FP1	25 – 30	25 – 30	0 – 5	5 – 10	5 – 10
Further Pure Mathematics FP2	25 – 30	25 – 30	0 – 5	7 – 12	5 – 10
Further Pure Mathematics FP3	25 – 30	25 – 30	0 – 5	7 – 12	5 – 10
Mechanics M1	20 – 25	20 – 25	15 – 20	6 – 11	4 – 9
Mechanics M2	20 – 25	20 – 25	10 – 15	7 – 12	5 – 10
Mechanics M3	20 – 25	25 – 30	10 – 15	5 – 10	5 – 10
Mechanics M4	20 – 25	20 – 25	15 – 20	5 – 10	5 – 10
Mechanics M5	20 – 25	20 – 25	15 – 20	5 – 10	5 – 10
Statistics S1	20 – 25	20 – 25	15 – 20	5 – 10	5 – 10
Statistics S2	25 – 30	20 – 25	10 – 15	5 – 10	5 – 10
Statistics S3	25 – 30	20 – 25	10 – 15	5 – 10	5 – 10
Statistics S4	15 – 20	15 – 20	15 – 20	5 – 10	10 – 15
Decision Mathematics D1	20 – 25	20 – 25	15 – 20	5 – 10	5 – 10
Decision Mathematics D2	25 – 30	20 – 25	10 – 15	7 – 13	0 – 5

Notation

The following notation will be used in all mathematics examinations:

1. Set Notation

\in	is an element of
\notin	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements x_1, x_2, \dots
$\{x : \dots\}$	the set of all x such that ...
$n(A)$	the number of elements in set A
\emptyset	the empty set
\mathcal{E}	the universal set
A'	the complement of the set A
\mathbb{N}	the set of natural numbers, $\{1, 2, 3, \dots\}$
\mathbb{Z}	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
\mathbb{Z}^+	the set of positive integers, $\{1, 2, 3, \dots\}$
\mathbb{Z}_n	the set of integers modulo n , $\{0, 1, 2, \dots, n-1\}$
\mathbb{Q}	the set of rational numbers, $\left\{ \frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+ \right\}$
\mathbb{Q}^+	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
\mathbb{Q}_0^+	the set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
\mathbb{R}	the set of real numbers
\mathbb{R}^+	the set of positive real numbers $\{x \in \mathbb{R} : x > 0\}$
\mathbb{R}_0^+	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
\mathbb{C}	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the cartesian product of sets A and B , ie $A \times B = \{(a, b) : a \in A, b \in B\}$
\subseteq	is a subset of
\subset	is a proper subset of
\cup	union
\cap	intersection
$[a, b]$	the closed interval, $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b), [a, b [$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b],]a, b [$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
$(a, b),]a, b [$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y R x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x , in the context of some equivalence relation

2. Miscellaneous Symbols

$=$	is equal to
\neq	is not equal to
\equiv	is identical to or is congruent to
\approx	is approximately equal to
\cong	is isomorphic to
\propto	is proportional to
$<$	is less than
\leq, \nlessgtr	is less than or equal to, is not greater than
$>$	is greater than
\geq, \ngtr	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
\exists	there exists
\forall	for all

3. Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b, ab, a.b$	a multiplied by b
$a \div b, \frac{a}{b}, a/b$	a divided by b
$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
$\prod_{i=1}^n a_i$	$a_1 \times a_2 \times \dots \times a_n$
\sqrt{a}	the positive square root of a
$ a $	the modulus of a
$n!$	n factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n \in \mathbf{Z}^+$
	$\frac{n(n-1)\dots(n-r+1)}{r!}$ for $n \in \mathbf{Q}$

4. Functions

$f(x)$	the value of the function f at x
$f : A \rightarrow B$	f is a function under which each element of set A has an image in set B
$f : x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
$g \circ f, gf$	the composite function of f and g which is defined by $(g \circ f)(x)$ or $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as x tends to a
$\Delta x, \delta x$	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the n th derivative of y with respect to x
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ... , n th derivatives of $f(x)$ with respect to x
$\int y \, dx$	the indefinite integral of y with respect to x
$\int_a^b y \, dx$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
$\frac{\partial V}{\partial x}$	the partial derivative of V with respect to x
\dot{x}, \ddot{x}, \dots	the first, second, ... derivatives of x with respect to t

5. Exponential and Logarithmic Functions

e	base of natural logarithms
$e^x, \exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x, \log_e x$	natural logarithm of x
$\lg x, \log_{10} x$	logarithm of x to base 10

6. Circular and Hyperbolic Functions

$\left. \begin{array}{l} \sin, \cos, \tan, \\ \operatorname{cosec}, \sec, \cot \end{array} \right\}$	the circular functions
$\left. \begin{array}{l} \arcsin, \arccos, \arctan, \\ \operatorname{arccosec}, \operatorname{arcsec}, \operatorname{arccot} \end{array} \right\}$	the inverse circular functions
$\left. \begin{array}{l} \sinh, \cosh, \tanh, \\ \operatorname{cosech}, \operatorname{sech}, \operatorname{coth} \end{array} \right\}$	the hyperbolic functions
$\left. \begin{array}{l} \operatorname{arsinh}, \operatorname{arcosh}, \operatorname{artanh}, \\ \operatorname{arcosech}, \operatorname{arsech}, \operatorname{arcoth} \end{array} \right\}$	the inverse hyperbolic functions

7. Complex Numbers

i, j	square root of -1
z	a complex number, $z = x + iy$
$\operatorname{Re} z$	the real part of z , $\operatorname{Re} z = x$
$\operatorname{Im} z$	the imaginary part of z , $\operatorname{Im} z = y$
$ z $	the modulus of z , $ z = \sqrt{(x^2 + y^2)}$
$\arg z$	the argument of z , $\arg z = \theta$, $-\pi < \theta \leq \pi$
z^*	the complex conjugate of z , $x - iy$

8. Matrices

\mathbf{M}	a matrix \mathbf{M}
\mathbf{M}^{-1}	the inverse of the matrix \mathbf{M}
\mathbf{M}^T	the transpose of the matrix \mathbf{M}
$\det \mathbf{M}$ or $ \mathbf{M} $	the determinant of the square matrix \mathbf{M}

9. Vectors

\mathbf{a}	the vector \mathbf{a}
\vec{AB}	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of \mathbf{a}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$ \mathbf{a} , a$	the magnitude of \mathbf{a}
$\left \vec{AB} \right , AB$	the magnitude of \vec{AB}
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of \mathbf{a} and \mathbf{b}
$\mathbf{a} \times \mathbf{b}$	the vector product of \mathbf{a} and \mathbf{b}

10. Probability and Statistics

A, B, C , etc	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
$P(A)$	probability of the event A
A'	complement of the event A
$P(A B)$	probability of the event A conditional on the event B
X, Y, R , etc	random variables
x, y, r , etc	values of the random variables X, Y, R , etc
x_1, x_2, \dots	observations
f_1, f_2, \dots	frequencies with which the observations x_1, x_2, \dots occur
$p(x)$	probability function $P(X = x)$ of the discrete random variable X

p_1, p_2, \dots	probabilities of the values x_1, x_2, \dots of the discrete random variable X
$f(x), g(x), \dots$	the value of the probability density function of a continuous random variable X
$F(x), G(x), \dots$	the value of the (cumulative) distribution function $P(X \leq x)$ of a continuous random variable X
$E(X)$	expectation of the random variable X
$E[g(X)]$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable X
$G(t)$	probability generating function for a random variable which takes the values 0, 1, 2, ...
$B(n, p)$	binomial distribution with parameters n and p
$N(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
\bar{x}, m	sample mean
$s^2, \hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $N(0,1)$
Φ	corresponding cumulative distribution function
ρ	product moment correlation coefficient for a population
r	product moment correlation coefficient for a sample
$\text{Cov}(X, Y)$	covariance of X and Y

Specification content

Unit C1 – Core Mathematics

The examination

The examination will consist of one 1½ hour paper. It will contain about ten questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Formulae which candidates are expected to know are given in the appendix to this unit and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

For this unit, candidates may **not** have access to any calculating aids, including log tables and slide rules.

Preamble

Construction and presentation of rigorous mathematical arguments through appropriate use of precise statements and logical deduction, involving correct use of symbols and appropriate connecting language is required. Candidates are expected to exhibit correct understanding and use of mathematical language and grammar in respect of terms such as ‘equals’, ‘identically equals’, ‘therefore’, ‘because’, ‘implies’, ‘is implied by’, ‘necessary’, ‘sufficient’, and notation such as \therefore , \Rightarrow , \Leftarrow and \Leftrightarrow .

SPECIFICATION

NOTES

1. Algebra and functions

Laws of indices for all rational exponents.	The equivalence of $a^{m/n}$ and $\sqrt[n]{a^m}$ should be known.
Use and manipulation of surds.	Candidates should be able to rationalise denominators.
Quadratic functions and their graphs.	
The discriminant of a quadratic function.	
Completing the square. Solution of quadratic equations.	Solution of quadratic equations by factorisation, use of the formula and completing the square.
Simultaneous equations: analytical solution by substitution.	For example, where one equation is linear and one equation is quadratic
Solution of linear and quadratic inequalities.	For example, $ax + b > cx + d$, $px^2 + qx + r \geq 0$, $px^2 + qx + r < ax + b$.

Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation.

Candidates should be able to use brackets. Factorisation of polynomials of degree n , $n \leq 3$, eg $x^3 + 4x^2 + 3x$. The notation $f(x)$ may be used. (Use of the factor theorem is *not* required.)

Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations.

Functions to include simple cubic functions and the reciprocal function $y = k/x$ with $x \neq 0$.

Knowledge of the term asymptote is expected.

Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.

Candidates should be able to apply one of these transformations to any of the above functions [quadratics, cubics, reciprocal] and sketch the resulting graph.

Given the graph of any function $y = f(x)$ candidates should be able to sketch the graph resulting from one of these transformations.

2. Coordinate geometry in the (x, y) plane

Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$.

Conditions for two straight lines to be parallel or perpendicular to each other.

To include

(i) the equation of a line through two given points,

(ii) the equation of a line parallel (or perpendicular) to a given line through a given point. For example, the line perpendicular to the line $3x + 4y = 18$ through the point $(2, 3)$ has equation $y - 3 = \frac{4}{3}(x - 2)$.

3. Sequences and series

Sequences, including those given by a formula for the n th term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$.

Arithmetic series, including the formula for the sum of the first n natural numbers.

The general term and the sum to n terms of the series are required. The proof of the sum formula should be known.

Understanding of Σ notation will be expected.

4. Differentiation

The derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.

For example, knowledge that $\frac{dy}{dx}$ is the rate of change of y with respect to x . Knowledge of the chain rule is not required.

The notation $f'(x)$ may be used.

Differentiation of x^n , and related sums and differences.

E.g., for $n \neq 1$, the ability to differentiate expressions such as $(2x + 5)(x - 1)$ and $\frac{x^2 + 5x - 3}{3x^{1/2}}$ is expected.

Applications of differentiation to gradients, tangents and normals.

Use of differentiation to find equations of tangents and normals at specific points on a curve.

5. Integration

Indefinite integration as the reverse of differentiation.

Candidates should know that a constant of integration is required.

Integration of x^n .

For example, the ability to integrate expressions such as $\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}$, $\frac{(x+2)^2}{x^{\frac{1}{2}}}$ is expected.

Given $f'(x)$ and a point on the curve, candidates should be able to find an equation of the curve in the form $y = f(x)$.

Appendix C1 – Formulae

This appendix lists formulae that candidates are expected to remember and that may not be included in formulae booklets.

Quadratic equations

$$ax^2 + bx + c = 0 \text{ has roots } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

function	derivative
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x^n	nx^{n-1}
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Integration

function	integral
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x^n	$\frac{1}{n+1}x^{n+1} + c, n \neq -1$
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Unit C2 – Core Mathematics

The examination will consist of one 1½ hour paper. It will contain about nine questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.

Formulae which candidates are expected to know are given in the appendix to this unit and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Prerequisites

A knowledge of the specification for C1, its preamble and its associated formulae, is assumed and may be tested.

SPECIFICATION

NOTES

1. Algebra and functions

Simple algebraic division; use of the Factor Theorem and the Remainder Theorem.

Only division by $(x + a)$ or $(x - a)$ will be required.

Candidates should know that if $f(x) = 0$ when $x = a$, then $(x - a)$ is a factor of $f(x)$.

Candidates may be required to factorise cubic expressions such as $x^3 + 3x^2 - 4$ and $6x^3 + 11x^2 - x - 6$.

Candidates should be familiar with the terms 'quotient' and 'remainder' and be able to determine the remainder when the polynomial $f(x)$ is divided by $(ax + b)$.

2. Coordinate geometry in the (x, y) plane

Coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$ and including use of the following circle properties:

- (i) the angle in a semicircle is a right angle;
- (ii) the perpendicular from the centre to a chord bisects the chord;
- (iii) the perpendicularity of radius and tangent.

Candidates should be able to find the radius and the coordinates of the centre of the circle given the equation of the circle, and vice versa.

3. Sequences and series

The sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $|r| < 1$.

The general term and the sum to n terms are required.

The proof of the sum formula should be known.

Binomial expansion of $(1+x)^n$ for positive integer n . The notations $n!$ and $\binom{n}{r}$.

Expansion of $(a+bx)^n$ may be required.

4. Trigonometry

The sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab \sin C$.

Radian measure, including use for arc length and area of sector.

Use of the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ for a circle.

Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.

Knowledge of graphs of curves with equations such as $y = 3 \sin x$, $y = \sin(x + \pi/6)$, $y = \sin 2x$ is expected.

Knowledge and use of $\tan \theta = \frac{\sin \theta}{\cos \theta}$, and $\sin^2 \theta + \cos^2 \theta = 1$.

Solution of simple trigonometric equations in a given interval.

Candidates should be able to solve equations such as

$$\sin(x - \pi/2) = \frac{3}{4} \text{ for } 0 < x < 2\pi,$$

$$\cos(x + 30^\circ) = \frac{1}{2} \text{ for } -180^\circ < x < 180^\circ,$$

$$\tan 2x = 1 \text{ for } 90^\circ < x < 270^\circ,$$

$$6 \cos^2 x^\circ + \sin x^\circ - 5 = 0 \text{ for } 0 \leq x < 360,$$

$$\sin^2\left(x + \frac{\pi}{6}\right) = \frac{1}{2} \text{ for } -\pi \leq x < \pi.$$

5. Exponentials and logarithms

$y = a^x$ and its graph.

Laws of logarithms

To include

$$\log_a xy = \log_a x + \log_a y,$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y,$$

$$\log_a x^k = k \log_a x,$$

$$\log_a \frac{1}{x} = -\log_a x,$$

$$\log_a a = 1.$$

The solution of equations of the form $a^x = b$.

Candidates may use the change of base formula.

6. Differentiation

Applications of differentiation to maxima and minima and stationary points, increasing and decreasing functions.

The notation $f''(x)$ may be used for the second order derivative.

To include applications to curve sketching. Maxima and minima problems may be set in the context of a practical problem.

7. Integration

Evaluation of definite integrals.

Interpretation of the definite integral as the area under a curve.

Candidates will be expected to be able to evaluate the area of a region bounded by a curve and given straight lines.

E.g. find the finite area bounded by the curve $y = 6x - x^2$ and the line $y = 2x$.

$\int x \, dy$ will not be required.

Approximation of area under a curve using the trapezium rule.

E.g. evaluate $\int_0^1 \sqrt{2x+1} \, dx$ using the values of $\sqrt{2x+1}$ at $x = 0, 0.25, 0.5, 0.75$ and 1.

Appendix C2 – Formulae

This appendix lists formulae that candidates are expected to remember and that may not be included in formulae booklets.

Laws of logarithms

$$\log_a x + \log_a y \equiv \log_a (xy)$$

$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y} \right)$$

$$k \log_a x \equiv \log_a (x^k)$$

Trigonometry

In the triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{area} = \frac{1}{2} ab \sin C$$

Area

$$\text{area under a curve} = \int_a^b y \, dx \quad (y \geq 0)$$

Unit C3 – Core Mathematics

The examination

The examination will consist of one 1½ hour paper. It will contain about seven questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.

Formulae which candidates are expected to know are given in the appendix to this unit and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Prerequisites

A knowledge of the specification for C1 and C2, their preambles, prerequisites and associated formulae, is assumed and may be tested.

Preamble

Methods of proof, including proof by contradiction and disproof by counter-example, are required. At least one question on the paper will require the use of proof.

SPECIFICATION

NOTES

1. Algebra and functions

- | | | |
|---|--|--|
| S | Simplification of rational expressions including factorising and cancelling, and algebraic division. | Denominators of rational expressions will be linear or quadratic, eg $\frac{1}{ax+b}$, $\frac{ax+b}{px^2+qx+r}$,
$\frac{x^3+1}{x^2-1}$. |
| S | Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs. | The concept of a function as a one-one or many-one mapping from \mathbb{R} (or a subset of \mathbb{R}) to \mathbb{R} . The notation $f : x \mapsto \dots$ and $f(x)$ will be used.

Candidates should know that fg will mean ‘do g first, then f ’.

Candidates should know that if f^{-1} exists, then $f^{-1}f(x) = ff^{-1}(x) = x$. |

S The modulus function. Candidates should be able to sketch the graphs of $y = |ax + b|$ and the graphs of $y = |f(x)|$ and $y = f(|x|)$, given the graph of $y = f(x)$.

S Combinations of the transformations $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$. Candidates should be able to sketch the graph of, e.g., $y = 2f(3x)$, $y = f(-x) + 1$, given the graph of $y = f(x)$ or the graph of, e.g. $y = 3 + \sin 2x$, $y = -\cos(x + \frac{\pi}{4})$.

The graph of $y = f(ax + b)$ will *not* be required.

2. Trigonometry

S Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains. Angles measured in both degrees and radians.

S Knowledge and use of $\sec^2 \theta = 1 + \tan^2 \theta$ and $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$.

S Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a \cos \theta + b \sin \theta$ in the equivalent forms of $r \cos(\theta \pm a)$ or $r \sin(\theta \pm a)$. To include application to half-angles. Knowledge of the t ($\tan \frac{1}{2}\theta$) formulae will *not* be required.

Candidates should be able to solve equations such as $a \cos \theta + b \sin \theta = c$ in a given interval, and to prove simple identities such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$.

3. Exponentials and logarithms

N The function e^x and its graph. To include the graph of $y = e^{ax+b} + c$.

N The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x . Solution of equations of the form $e^{ax+b} = p$ and $\ln(ax + b) = q$ is expected.

4. Differentiation

- S Differentiation of e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$ and their sums and differences.
- S Differentiation using the product rule, the quotient rule and the chain rule. Differentiation of $\operatorname{cosec} x$, $\cot x$ and $\sec x$ are required. Skill will be expected in the differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, $\frac{e^{3x}}{x}$, $\cos x^2$ and $\tan^2 2x$.

The use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.

E.g. finding $\frac{dy}{dx}$ for $x = \sin 3y$.

5. Numerical methods

- N Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.
- N Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$. Solution of equations by use of iterative procedures for which leads will be given

Appendix C3 – Formulae

This appendix lists formulae that candidates are expected to remember and that may not be included in formulae booklets.

Trigonometry

$$\cos^2 A + \sin^2 A \equiv 1$$

$$\sec^2 A \equiv 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

Differentiation

function	derivative
$\sin kx$	$k \cos kx$
$\cos kx$	$-k \sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Unit C4 – Core Mathematics

The examination

The examination will consist of one 1½ hour paper. It will contain about seven questions of varying length. The mark allocations per question which will be stated on the paper. All questions should be attempted.

Formulae which candidates are expected to know are given in the appendix to this unit and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Prerequisites

A knowledge of the specification for C1, C2 and C3 and their preambles, prerequisites and associated formulae, is assumed and may be tested.

SPECIFICATION

NOTES

1. Algebra and functions

- S Rational functions. Partial fractions (denominators not more complicated than repeated linear terms). Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$. The degree of the numerator may equal or exceed the degree of the denominator. Applications to integration, differentiation and series expansions.
- Quadratic factors in the denominator such as $(x^2 + a)$, $a > 0$, are *not* required.

2. Coordinate geometry in the (x, y) plane

- S Parametric equations of curves and conversion between Cartesian and parametric forms. Candidates should be able to find the area under a curve given its parametric equations. Candidates will *not* be expected to sketch a curve from its parametric equations.

3. Sequences and series

- S Binomial series for any rational n . For $|x| < \frac{b}{a}$, candidates should be able to obtain the expansion of $(ax + b)^n$, and the expansion of rational functions by decomposition into partial fractions.

4. Differentiation

- S Differentiation of simple functions defined implicitly or parametrically. The finding of equations of tangents and normals to curves given parametrically or implicitly is required.
- S Exponential growth and decay. Knowledge and use of the result $\frac{d}{dx}(a^x) = a^x \ln a$ is expected.
- S Formation of simple differential equations. Questions involving connected rates of change may be set.

5. Integration

- S Integration of e^x , $\frac{1}{x}$, $\sin x$, $\cos x$. To include integration of standard functions such as $\sin 3x$, $\sec^2 2x$, $\tan x$, e^{5x} , $\frac{1}{2x}$.
Candidates should recognise integrals of the form $\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$.
Candidates are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$.
- S Evaluation of volume of revolution. $\pi \int y^2 dx$ is required but *not* $\pi \int x^2 dy$. Candidates should be able to find a volume of revolution, given parametric equations.
- S Simple cases of integration by substitution and integration by parts. These methods as the reverse processes of the chain and product rules respectively. Except in the simplest of cases the substitution will be given. The integral $\int \ln x dx$ is required. More than one application of integration by parts may be required, for example $\int x^2 e^x dx$.
- S Simple cases of integration using partial fractions. Integration of rational expressions such as those arising from partial fractions, e.g. $\frac{2}{3x+5}$, $\frac{3}{(x-1)^2}$.
Note that the integration of other rational expressions, such as $\frac{x}{x^2+5}$ and $\frac{2}{(2x-1)^4}$ is also required (see above paragraphs).
- S Analytical solution of simple first order differential equations with separable variables. General and particular solutions will be required.

- S Numerical integration of functions. Application of the trapezium rule to functions covered in C3 and C4. Use of increasing number of trapezia to improve accuracy and estimate error will be required. Questions will not require more than three iterations.

Simpson's Rule is *not* required.

6. Vectors

- N Vectors in two and three dimensions.
- N Magnitude of a vector. Candidates should be able to find a unit vector in the direction of \mathbf{a} , and be familiar with $|\mathbf{a}|$.
- N Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.
- N Position vectors. The distance between two points. $\vec{OB} - \vec{OA} = \vec{AB} = \mathbf{b} - \mathbf{a}$.
- The distance d between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$.
- N Vector equations of lines. To include the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + t(\mathbf{d} - \mathbf{c})$.
- Intersection, or otherwise, of two lines.
- N The scalar product. Its use for calculating the angle between two lines. Candidates should know that for $\vec{OA} = \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\vec{OB} = \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ and $\cos \angle AOB = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$.
- Candidates should know that if $\mathbf{a} \cdot \mathbf{b} = 0$, and that \mathbf{a} and \mathbf{b} are non-zero vectors, then \mathbf{a} and \mathbf{b} are perpendicular.

Appendix C4 – Formulae

This appendix lists formulae that candidates are expected to remember and that may not be included in formulae booklets.

Integration

function	integral
$\cos kx$	$\frac{1}{k} \sin kx + c$
$\sin kx$	$-\frac{1}{k} \cos kx + c$
e^{kx}	$\frac{1}{k} e^{kx} + c$
$\frac{1}{x}$	$\ln x + c, x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))g'(x)$	$f(g(x)) + c$

Vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$$

Unit FP1 – Further Pure Mathematics

The examination

The examination will consist of one 1½ hour paper. It will contain about eight questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Prerequisites

A knowledge of the specifications for C1, C2, C3 and C4 is assumed and may be tested.

SPECIFICATION

NOTES

1. Inequalities

The manipulation and solution of algebraic inequalities and inequations, including those involving the modulus sign.

The solution of inequalities such as $\frac{1}{x-a} > \frac{x}{x-b}$, $|x^2 - 1| > 2(x + 1)$.

2. Series

Summation of simple finite series. The method of differences.

Candidates should be able to sum series such as $\sum_{r=1}^n r$, $\sum_{r=1}^n r^2$, $\sum_{r=1}^n r(r^2 + 2)$.

Proof by induction is not required.

3. Complex numbers

Definition of complex numbers in the form $a + ib$ and $r \cos \theta + i r \sin \theta$.

The meaning of conjugate, modulus, argument, real part, imaginary part and equality of complex numbers should be known.

Sum, product and quotient of complex numbers.

Geometrical representation of complex numbers in the Argand diagram. Geometrical representation of sums, products and quotients of complex numbers.

Complex solutions of quadratic equations with real coefficients.

Conjugate complex roots of polynomial equations with real coefficients.

Knowledge that if z_1 is a root of $f(z) = 0$ then z_1^* is also a root.

4. Numerical solution of equations

Equations of the form $f(x) = 0$ solved numerically by

- (i) interval bisection,
- (ii) linear interpolation,
- (iii) the Newton-Raphson process.

5. First Order Differential Equations

Further solution of first order differential equations with separable variables.

First order linear differential equations of the form $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x .

Differential equations reducible to the above types by means of a given substitution.

6. Second Order Differential Equations

The linear second order differential equation $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ where a , b and c are real constants and the particular integral can be found by inspection or trial.

Differential equations reducible to the above types by means of a given substitution.

The formation of the differential equation may be required. Candidates will be expected to obtain particular solutions and also sketch members of the family of solution curves.

The integrating factor $e^{\int P dx}$ may be quoted without proof.

The auxiliary equation may have real distinct, equal or complex roots. $f(x)$ will have one of the forms ke^{px} , $A + Bx$, $p + qx + cx^2$ or $m \cos \omega x + n \sin \omega x$. Candidates should be familiar with the terms 'complementary function' and 'particular integral'.

Candidates should be able to solve equations of the form $\frac{d^2y}{dx^2} + 4y = \sin 2x$.

7. Polar Coordinates

Polar coordinates (r, θ) , $r \geq 0$.

The sketching of curves such as $\theta = \alpha$,
 $r = p \sec(\alpha - \theta)$, $r = a$, $r = 2a \cos \theta$, $r = k\theta$,
 $r = a(1 \pm \cos \theta)$, $r = a(3 + 2 \cos \theta)$, $r = a \cos 2\theta$ and $r^2 = a^2 \cos 2\theta$ may be set.

The ability to find tangents parallel to, or at right angles to, the initial line is expected.

Use of the formula $\frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$ for area.

Unit FP2 – Further Pure Mathematics

The examination

The examination will consist of one 1½ hour paper. It will contain about eight questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Prerequisites

A knowledge of the specifications for C1, C2, C3, C4 and FP1 and their associated formulae is assumed and may be tested.

SPECIFICATION

NOTES

1. Coordinate Systems

S*	Cartesian and parametric equations for the parabola, ellipse, hyperbola and rectangular hyperbola.	Candidates should be familiar with the equations: $y^2 = 4ax; x = at^2, y = 2at.$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; x = a \cos t, y = b \sin t.$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; x = a \sec t, y = b \tan t$ and $x = a \cosh t, y = b \sinh t.$ $xy = c^2; x = ct, y = \frac{c}{t}.$
S*	The focus-directrix properties of the parabola, ellipse and hyperbola, including the eccentricity.	For example, candidates should know that, for the ellipse, $b^2 = a^2(1 - e^2)$, the foci are $(ae, 0)$ and $(-ae, 0)$ and the equations of the directrices are $x = +\frac{a}{e}$ and $x = -\frac{a}{e}$.
S*	Tangents and normals to these curves.	The condition for $y = mx + c$ to be a tangent to these curves is expected to be known.
S*	Simple loci problems.	

- S* Intrinsic coordinates (s, ψ) . Candidates should be familiar with the equations $\frac{dy}{dx} = \tan \psi$, $\frac{dx}{ds} = \cos \psi$, $\frac{dy}{ds} = \sin \psi$. In appropriate cases, candidates should be able to obtain intrinsic equations of curves given cartesian or parametric equations.
- S* Radius of curvature. For curves with cartesian, parametric or intrinsic equations.

2. Hyperbolic functions

- N Definition of the six hyperbolic functions in terms of exponentials. Graphs and properties of the hyperbolic functions. For example, $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$. Candidates should be able to derive and use simple identities such as $\cosh^2 x - \sinh^2 x \equiv 1$ and $\cosh^2 x + \sinh^2 x \equiv \cosh 2x$ and to solve equations such as $a \cosh x + b \sinh x = c$.
- N Inverse hyperbolic functions, their graphs, properties and logarithmic equivalents. For example, $\operatorname{arsinh} x = \ln[x + \sqrt{1+x^2}]$ and candidates may be required to prove this and similar results.

3. Differentiation

- S* Differentiation of hyperbolic functions and expressions involving them. For example, $\tanh 3x$, $x \sinh^2 x$, $\frac{\cosh 2x}{\sqrt{x+1}}$.
- S* Differentiation of inverse functions, including trigonometric and hyperbolic functions. For example, $\arcsin x + x\sqrt{1-x^2}$, $\frac{1}{2}(\operatorname{artanh} x^2)$.

4. Integration

- S* Integration of hyperbolic functions and expressions involving them.
- S* Integration of inverse trigonometric and hyperbolic functions. For example, $\int \operatorname{arsinh} x \, dx$, $\int \arctan x \, dx$.
- S* Integration using hyperbolic and trigonometric substitutions. To include the integrals of $1/(a^2 + x^2)$, $1/\sqrt{a^2 - x^2}$, $1/\sqrt{a^2 + x^2}$, $1/\sqrt{x^2 - a^2}$.
- S* Use of substitution for integrals involving quadratic surds. In more complicated cases, substitutions will be given.

S
* The derivation and use of simple reduction formulae.

Candidates should be able to derive formulae such as

$$nI_n = (n - 1)I_{n - 2}, \quad n \geq 2, \quad \text{for } I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx,$$

$$I_{n+2} = \frac{2 \sin(n+1)x}{n+1} + I_n \quad \text{for } I_n = \int \frac{\sin nx}{\sin x} \, dx, \quad n > 0.$$

S
* The calculation of arc length and the area of a surface of revolution.

The equation of the curve may be given in cartesian or parametric form. Equations in polar or intrinsic form will not be set.

Unit FP3 – Further Pure Mathematics

The examination

The examination will consist of one 1½ hour paper. It will contain about eight questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π, x², √x, $\frac{1}{x}$, x^y, ln x, e^x, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Prerequisites

A knowledge of the specifications for C1, C2, C3, FP1 and their associated formulae, plus hyperbolic functions, is assumed and may be tested.

SPECIFICATION

NOTES

1. Complex Numbers

S*	Euler's relation $e^{i\theta} = \cos \theta + i \sin \theta$.	Candidates should be familiar with
S*	Relations between trigonometric functions and hyperbolic functions.	$\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ and $\sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$
N	De Moivre's theorem and its application to trigonometric identities and to roots of a complex number.	To include finding $\cos n\theta$ and $\sin m\theta$ in terms of powers of $\sin \theta$ and $\cos \theta$ and also powers of $\sin \theta$ and $\cos \theta$ in terms of multiple angles. Candidates should be able to prove De Moivre's theorem for any integer n .
N	Loci and regions in the Argand diagram.	Loci such as $ z - a = b$, $ z - a = k z - b $, $\arg(z - a) = \beta$ and $\arg \frac{z - a}{z - b} = \beta$ and regions such as $ z - a \leq z - b $, $ z - a \leq b$.
N	Elementary transformations from the z -plane to the w -plane.	Transformations such as $w = z^2$ and $w = \frac{az + b}{cz + d}$, where $a, b, c, d \in \mathbb{C}$, may be set.

2. Matrix Algebra

N	Linear transformations of column vectors in two and three dimensions and their matrix representation. Combination of	The transformation represented by AB is the transformation represented by B followed by the transformation represented by A .
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transformations. Products of matrices.

Applications of matrices to geometrical transformations.

Use of the relation $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$.

Singular and non-singular matrices.

Use of the relation $(\mathbf{AB})^{-1} = \mathbf{B}^{-1} \mathbf{A}^{-1}$.

Normalised vectors may be required.

Candidates should be able to find an orthogonal matrix \mathbf{P} such that $\mathbf{P}^T \mathbf{A} \mathbf{P}$ is diagonal.

3. Vectors

The interpretation of $|\mathbf{a} \times \mathbf{b}|$ as an area and $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ as a volume.

Candidates may be required to use equivalent cartesian forms also. Applications to include
(i) distance from a point to a plane,
(ii) line of intersection of two planes,
(iii) shortest distance between two skew lines.

Candidates may be required to use equivalent cartesian forms also.

4. Maclaurin and Taylor series

The derivation of the series expansion of e^x , $\sin x$, $\cos x$, $\ln(1+x)$ and other simple functions may be required.

The derivation, for example, of the expansion of $\sin x$ in ascending powers of $(x - \pi)$ up to and including the term in $(x - \pi)^3$.

Candidates may, for example, be required to find the solution in powers of x as far as the term in x^4 , of the differential equation

$$\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$
, such that $y = 1$, $\frac{dy}{dx} = 0$ at $x = 0$.

5. Numerical Methods

N Numerical solution of first and second order differential equations by step-by-step methods.

The approximations

$$\left(\frac{dy}{dx}\right)_0 \approx (y_1 - y_0)/h,$$

$$\left(\frac{dy}{dx}\right)_0 \approx (y_1 - y_{-1})/2h,$$

$$\left(\frac{d^2y}{dx^2}\right)_0 \approx (y_1 - 2y_0 + y_{-1})/h^2,$$

where appropriate will be given, but the derivation of these results may be required.

6. Proof

S* Proof by mathematical induction.

Candidates should be able to prove De Moivre's theorem and the binomial theorem. To include induction proofs for

(i) summation of series,

$$\text{eg show } \sum_{r=1}^n r^3 = \frac{1}{4} n^2(n+1)^2,$$

(ii) divisibility, eg show $3^{2n} + 11$ is divisible by 4,

(iii) inequalities, eg

show $(1+x)^n > 1+nx$ for $n \geq 2, x > -1, x \neq 0$,

(iv) finding general terms in a sequence,

eg if $u_{n+2} = 5u_{n+1} - 6u_n$ with $u_1 = 1, u_2 = 5$, prove that $u_n = 3^n - 2^n$.

Unit M1 – Mechanics

The examination

The examination will consist of one 1½ hour paper. It will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Candidates will be expected to know and be able to recall and use the following formulae:

$$\text{Momentum} = mv$$

$$\text{Impulse} = mv - mu$$

For constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

Prerequisites

Candidates are expected to have a knowledge of C1 and of vectors in two dimensions.

SPECIFICATION

NOTES

1. Mathematical Models in Mechanics

The basic ideas of mathematical modelling as applied in Mechanics.

Candidates should be familiar with the terms: particle, lamina, rigid body, rod (light, uniform, non-uniform), inextensible string, smooth and rough surface, light smooth pulley, bead, wire, peg. Candidates should be familiar with the assumptions made in using these models.

2. Vectors in Mechanics

Magnitude and direction of a vector. Resultant of vectors may also be required.

Application of vectors to displacements, velocities, accelerations and forces in a plane.

Candidates may be required to resolve a vector into two components or use a vector diagram. Questions may be set involving the unit vectors \mathbf{i} and \mathbf{j} .

Use of velocity = $\frac{\text{change of displacement}}{\text{time}}$ in the case of constant velocity, and of acceleration = $\frac{\text{change of velocity}}{\text{time}}$ in the case of constant acceleration, will be required.

3. Kinematics of a particle moving in a straight line

Motion in a straight line with constant acceleration.

Graphical solutions may be required, including displacement-time, velocity-time, speed-time and acceleration-time graphs. Knowledge and use of formulae for constant acceleration will be required.

4. Dynamics of a particle moving in a straight line or plane

The concept of a force. Newton's laws of motion.

Simple problems involving constant acceleration in scalar form or as a vector of the form $a\mathbf{i} + b\mathbf{j}$.

Simple applications including the motion of two connected particles.

Problems may include
(i) the motion of two connected particles moving in a straight line or under gravity when the forces on each particle are constant; problems involving smooth fixed pulleys and/or pegs may be set;
(ii) motion under a force which changes from one fixed value to another, eg a particle hitting the ground;
(iii) motion directly up or down a smooth or rough inclined plane.

Momentum and impulse. The impulse-momentum principle. The principle of conservation of momentum applied to two particles colliding directly.

Knowledge of Newton's law of restitution is not required. Problems will be confined to those of a one-dimensional nature.

Coefficient of friction.

An understanding of $F = \mu R$ when a particle is moving.

5. Statics of a particle

Forces treated as vectors. Resolution of forces.

Equilibrium of a particle under coplanar forces. Weight, normal reaction, tension and thrust, friction.

Coefficient of friction.

Only simple cases of the application of the conditions for equilibrium to uncomplicated systems will be required.

An understanding of $F \leq \mu R$ in a situation of equilibrium.

6. Moments

Moment of a force.

Simple problems involving coplanar parallel forces acting on a body and conditions for equilibrium in such situations.

Unit M2 – Mechanics

The examination

The examination will consist of one 1½ hour paper. It will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Candidates will be expected to know and be able to recall and use the following formulae:

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

$$\text{Potential energy} = mgh$$

Prerequisites

A knowledge of the specification for M1 and its prerequisites and associated formulae, together with a knowledge of algebra, trigonometry, differentiation and integration, as specified in C1, C2 and C3, is assumed and may be tested.

SPECIFICATION

NOTES

1. Kinematics of a particle moving in a straight line or plane

S	Motion in a vertical plane with constant acceleration, eg under gravity.	
S	Simple cases of motion of a projectile.	
S	Velocity and acceleration when the displacement is a function of time.	The setting up and solution of equations of the form $\frac{dx}{dt} = f(t)$ or $\frac{dv}{dt} = g(t)$ will be consistent with the level of calculus in C2.
S	Differentiation and integration of a vector with respect to time.	For example, given that $\mathbf{r} = t^2\mathbf{i} + t^{3/2}\mathbf{j}$, find $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ at a given time.

2. Centres of mass

N Centre of mass of a discrete mass distribution in one and two dimensions.

N Centre of mass of uniform plane figures, and simple cases of composite plane figures.

The use of an axis of symmetry will be acceptable where appropriate. Use of integration is not required. Figures may include the shapes referred to in the formulae book. Results given in the formulae book may be quoted without proof.

S Simple cases of equilibrium of a plane lamina.

The lamina may
(i) be suspended from a fixed point;
(ii) free to rotate about a fixed horizontal axis;
(iii) be put on an inclined plane.

3. Work and energy

N Kinetic and potential energy, work and power. The work-energy principle. The principle of conservation of mechanical energy.

Problems involving motion under a constant resistance and/or up and down an inclined plane may be set.

4. Collisions

S Momentum as a vector. The impulse-momentum principle in vector form. Conservation of linear momentum.

N Direct impact of elastic particles. Newton's law of restitution. Loss of mechanical energy due to impact.

Candidates will be expected to know and use the inequalities $0 \leq e \leq 1$ (where e is the coefficient of restitution).

N Successive impacts of up to three particles or two particles and a smooth plane surface.

Collision with a plane surface will not involve oblique impact.

5. Statics of rigid bodies

S Moment of a force.

S Equilibrium of rigid bodies.

Problems involving parallel and non-parallel coplanar forces. Problems may include rods or ladders resting against smooth or rough vertical walls and on smooth or rough ground.

Unit M3 – Mechanics

The examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Candidates will be expected to know and be able to recall and use the following formulae:

The tension in an elastic string = $\frac{\lambda x}{l}$

The energy stored in an elastic string = $\frac{\lambda x^2}{2l}$

For SHM:

$$\ddot{x} = -\omega^2 x,$$

$$x = a \cos \omega t \text{ or } x = a \sin \omega t,$$

$$v^2 = \omega^2(a^2 - x^2),$$

$$T = \frac{2\pi}{\omega}$$

Prerequisites

A knowledge of the specifications for M1 and M2 and their prerequisites and associated formulae, together with a knowledge of differentiation, integration and differential equations, as specified in C1, C2, C3 and C4, is assumed and may be tested.

SPECIFICATION

NOTES

1. Further kinematics

Kinematics of a particle moving in a straight line when the acceleration is a function of the displacement (x), or time (t).

The setting up and solution of equations where $\frac{dv}{dt} = f(t)$, $v \frac{dv}{dx} = f(x)$, $\frac{dx}{dt} = f(x)$ or $\frac{dx}{dt} = f(t)$ will be consistent with the level of calculus required in units C1, C2 and C3.

2. Elastic strings and springs

Elastic strings and springs. Hooke's law.

Energy stored in an elastic string or spring.

Simple problems using the work-energy principle involving kinetic energy, potential energy and elastic energy.

3. Further dynamics

Newton's laws of motion, for a particle moving in one dimension, when the applied force is variable.

The solution of the resulting equations will be consistent with the level of calculus in units C2 and C3. Problems may involve the law of gravitation, ie the inverse square law.

Simple harmonic motion.

Proof that a particle moves with simple harmonic motion in a given situation may be required (ie showing that $\ddot{x} = -\omega^2 x$).

Geometric or calculus methods of solution will be acceptable. Candidates will be expected to be familiar with standard formulae, which may be quoted without proof.

Oscillations of a particle attached to the end of an elastic string or spring.

Oscillations will be in the direction of the string or spring only.

4. Motion in a circle

Angular speed.

Radial acceleration in circular motion.

The forms $r\omega^2$ and $\frac{v^2}{r}$ are required.

Uniform motion of a particle moving in a horizontal circle.

Problems involving the 'conical pendulum', an elastic string, motion on a banked surface, as well as other contexts, may be set.

Motion of a particle in a vertical circle.

5. Statics of rigid bodies

Centre of mass of uniform rigid bodies and simple composite bodies.

The use of integration and/or symmetry to determine the centre of mass of a uniform body will be required.

Simple cases of equilibrium of rigid bodies.

To include
(i) suspension of a body from a fixed point,
(ii) a rigid body placed on a horizontal or inclined plane.

Unit M4 – Mechanics

The examination

The examination will consist of one 1½ hour paper. It will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Candidates will be expected to know and be able to recall and use the following formulae:

$$A\mathbf{v}_B = \mathbf{v}_A - \mathbf{v}_B$$

Prerequisites

A knowledge of the specifications for M1, M2 and M3 and their prerequisites and associated formulae, together with a knowledge of the calculus covered in FP1 and $\int \frac{1}{a^2 + x^2} dx$, is assumed and may be tested.

SPECIFICATION

NOTES

1. Relative motion

Relative motion of two particles, including relative displacement and relative velocity.

Problems may be set in vector form and may involve problems of interception or closest approach including the determination of course required for closest approach.

2. Elastic collisions in two dimensions

Oblique impact of smooth elastic spheres and a smooth sphere with a fixed surface.

3. Further motion of particles in one dimension

Resisted motion of a particle moving in a straight line.

The resisting forces may include the forms $a + bv$ and $a + bv^2$ where a and b are constants and v is the speed.

Damped and/or forced harmonic motion.

The damping to be proportional to the speed. Solution of the relevant differential equations will be expected.

4. Stability

Finding equilibrium positions of a system from consideration of its potential energy.

Positions of stable and unstable equilibrium of a system.

Unit M5 – Mechanics

The examination

The examination will consist of one 1½ hour paper. It will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Candidates will be expected to know and be able to recall and use the following formulae:

Work done by a constant force = $\mathbf{F} \cdot \mathbf{d}$

Angular momentum = $I\dot{\theta}$

Rotational kinetic energy = $\frac{1}{2}I\dot{\theta}^2$

$L = I\ddot{\theta}$

$\int_{t_1}^{t_2} L \, dt = I\omega_2 - I\omega_1$

For constant angular acceleration:

$$\omega_1 = \omega_0 + \alpha t$$

$$\omega_1^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = \frac{(\omega_0 + \omega_1)}{2} t$$

Prerequisites

A knowledge of the specifications for M1, M2, M3 and M4 and their prerequisites and associated formulae, together with a knowledge of scalar and vector products, and of differential equations as specified in FP1, is assumed and may be tested.

SPECIFICATION

1. Applications of vectors in mechanics

Solution of simple vector differential equations.

Work done by a constant force using a scalar product.

Moment of a force using a vector product.

The analysis of simple systems of forces in three dimensions acting on a rigid body.

2. Variable mass

Motion of a particle with varying mass.

3. Moments of inertia of a rigid body

Moments of inertia and radius of gyration of standard and composite bodies.

The parallel and perpendicular axes theorems.

4. Rotation of a rigid body about a fixed smooth axis

Motion of a rigid body about a fixed smooth horizontal or vertical axis.

NOTES

Vector differential equations such as

$$\frac{d\mathbf{v}}{dt} = k\mathbf{v}, \quad \frac{d^2\mathbf{r}}{dt^2} + 2k\frac{d\mathbf{r}}{dt} + (k^2 + n^2)\mathbf{r} = \mathbf{f}(t)$$

where k and n are constants, $\frac{d\mathbf{r}}{dt} + 4\mathbf{r} = \mathbf{i}e^t$.

Use of an integrating factor may be required.

The moment of a force \mathbf{F} about O is defined as $\mathbf{r} \times \mathbf{F}$, where \mathbf{r} is the position vector of the point of application of \mathbf{F} .

The reduction of a system of forces acting on a body to a single force, single couple or a couple and a force acting through a stated point.

Candidates may be required to derive an equation of motion from first principles by considering the change in momentum over a small time interval.

Use of integration including the proof of the standard results given in the formulae booklet will be required.

Use of conservation of energy will be required. Calculation of the force on the axis will be required.

Angular momentum.

Kinetic energy.

Conservation of angular momentum.
The effect of an impulse on a rigid body
which is free to rotate about a fixed axis.

Simple pendulum and compound
pendulum.

Unit S1 – Statistics

The examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Candidates will be expected to know and be able to recall and use the following formulae:

$$\text{Mean} = \bar{x} = \frac{\sum x}{n} \text{ or } \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\text{Variance}}$$

$$\text{Interquartile range} = \text{IQR} = Q_3 - Q_1$$

$$P(A') = 1 - P(A)$$

$$\text{For independent events, } P(B|A) = P(B), P(A|B) = P(A), P(A \cap B) = P(A)P(B)$$

$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

Cumulative distribution function for a discrete random variable:

$$F(x_0) = P(X \leq x_0) = \sum_{x \leq x_0} p(x)$$

$$\text{Standardised Normal Random Variable } Z = \frac{X - \mu}{\sigma} \text{ where } X \sim N(\mu, \sigma^2)$$

SPECIFICATION

NOTES

1. Mathematical models in probability and statistics

The basic ideas of mathematical modelling as applied in probability and statistics.

2. Representation and summary of data

Histograms, stem and leaf diagrams, box plots.

Use to compare distributions. Back-to-back stem and leaf diagrams may be required.

Histograms, stem and leaf diagrams, box plots will not be the direct focus of examination questions.

Measures of location – mean, median, mode.

Calculation of mean, mode and median, range and interquartile range will not be the direct focus of examination questions.

Candidates will be expected to draw simple inferences and give interpretations to measures of location and dispersion. Significance tests will not be expected.

Data may be discrete, continuous, grouped or ungrouped. Understanding and use of coding.

Measures of dispersion – variance, standard deviation, range and interpercentile ranges.

Simple interpolation may be required. Interpretation of measures of location and dispersion.

Skewness. Concepts of outliers.

Any rule to identify outliers will be specified in the question.

3. Probability

Elementary probability.
Sample space. Exclusive and complementary events. Conditional probability.

Understanding and use of
 $P(A') = 1 - P(A)$,
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$,
 $P(A \cap B) = P(A) P(B|A)$.

Independence of two events.

$P(B|A) = P(B)$, $P(A|B) = P(A)$,
 $P(A \cap B) = P(A) P(B)$.

Sum and product laws.

Use of tree diagrams and Venn diagrams. Sampling with and without replacement.

4. Correlation and regression

Scatter diagrams. Linear regression.

Calculation of the equation of a linear regression line using the method of least squares. Candidates may be required to draw this regression line on a scatter diagram.

Explanatory (independent) and response (dependent) variables. Applications and interpretations.

Use to make predictions within the range of values of the explanatory variable and the dangers of extrapolation. Derivations will not be required. Variables other than x and y may be used. Linear change of variable may be required.

The product moment correlation coefficient, its use, interpretation and limitations.

Derivations and tests of significance will not be required.

5. Discrete random variables

The concept of a discrete random variable.

The probability function and the cumulative distribution function for a discrete random variable.

Simple uses of the probability function $p(x)$ where $p(x) = P(X = x)$. Use of the cumulative distribution function

$$F(x_0) = P(X \leq x_0) = \sum_{x \leq x_0} p(x).$$

Mean and variance of a discrete random variable.

Use of $E(X)$, $E(X^2)$ for calculating the variance of X .

Knowledge and use of

$$E(aX + b) = aE(X) + b,$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X).$$

The discrete uniform distribution.

The mean and variance of this distribution.

6. The Normal distribution

The Normal distribution including the mean, variance and use of tables of the cumulative distribution function.

Knowledge of the shape and the symmetry of the distribution is required. Knowledge of the probability density function is not required. Derivation of the mean, variance and cumulative distribution function is not required. Interpolation is not necessary. Questions may involve the solution of simultaneous equations.

Unit S2 – Statistics

The examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Candidates will be expected to know and be able to recall and use the following formulae:

For the continuous random variable X having probability density function $f(x)$,

$$P(a < X \leq b) = \int_a^b f(x) dx .$$

$$f(x) = \frac{dF(x)}{dx} .$$

Prerequisites

A knowledge of the specification for S1 and its prerequisites and associated formulae, together with a knowledge of differentiation and integration of polynomials, binomial coefficients in connection with the binomial distribution and the evaluation of the exponential function is assumed and may be tested.

SPECIFICATION

NOTES

1. The Binomial and Poisson distributions

S	The binomial and Poisson distributions.	Candidates will be expected to use these distributions to model a real-world situation and to comment critically on their appropriateness. Cumulative probabilities by calculation or by reference to tables. Candidates will be expected to use the additive property of the Poisson distribution – eg if the number of events per minute $\sim \text{Po}(\lambda)$ then the number of events per 5 minutes $\sim \text{Po}(5\lambda)$.
S	The mean and variance of the binomial and Poisson distributions.	No derivations will be required.

N The use of the Poisson distribution as an approximation to the binomial distribution.

2. Continuous random variables

N The concept of a continuous random variable.

S The probability density function and the cumulative distribution function for a continuous random variable.

Use of the probability density function $f(x)$, where

$$P(a < X \leq b) = \int_a^b f(x) dx .$$

Use of the cumulative distribution function

$$F(x_0) = P(X \leq x_0) = \int_{-\infty}^{x_0} f(x) dx .$$

The formulae used in defining $f(x)$ will be restricted to simple polynomials which may be expressed piecewise.

S Relationship between density and distribution functions.

$$f(x) = \frac{dF(x)}{dx} .$$

S Mean and variance of continuous random variables.

S Mode, median and quartiles of continuous random variables.

3. Continuous distributions

N The continuous uniform (rectangular) distribution.

Including the derivation of the mean, variance and cumulative distribution function.

S Use of the Normal distribution as an approximation to the binomial distribution and the Poisson distribution, with the application of the continuity correction.

4. Hypothesis tests

N Population, census and sample. Sampling unit, sampling frame.

Candidates will be expected to know the advantages and disadvantages associated with a census and a sample survey.

N Concepts of a statistic and its sampling distribution.

N Concept and interpretation of a hypothesis test. Null and alternative hypotheses.

Use of hypothesis tests for refinement of mathematical models.

N Critical region.

Use of a statistic as a test statistic.

N One-tailed and two-tailed tests.

N Hypothesis tests for the parameter p of a binomial distribution and for the mean of a Poisson distribution.

Candidates are expected to know how to use tables to carry out these tests. Questions may also be set not involving tabular values. Tests on sample proportion involving the normal approximation will not be set.

Unit S3 – Statistics

The examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Candidates will be expected to know and be able to recall and use the following formulae:

$aX \pm bY \sim N(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2)$ where X and Y are independent and $X \sim N(\mu_x, \sigma_x^2)$ and $Y \sim N(\mu_y, \sigma_y^2)$.

Prerequisites

A knowledge of the specifications for S1 and S2 and their prerequisites and associated formulae is assumed and may be tested.

SPECIFICATION

NOTES

1. Combinations of random variables

Distribution of linear combinations of independent Normal random variables.

If $X \sim N(\mu_x, \sigma_x^2)$ and $Y \sim N(\mu_y, \sigma_y^2)$ independently, then
 $aX \pm bY \sim N(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2)$.
No proofs required.

2. Sampling

Methods for collecting data. Simple random sampling. Use of random numbers for sampling.

Other methods of sampling: stratified, systematic, quota.

The circumstances in which they might be used. Their advantages and disadvantages.

3. Estimation, confidence intervals and tests

Concepts of standard error, estimator, bias.

The sample mean, \bar{x} , and the sample variance, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$, as unbiased estimates of the corresponding population parameters.

The distribution of the sample mean \bar{X} .

\bar{X} has mean μ and variance $\frac{\sigma^2}{n}$.

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.

No proofs required.

Concept of a confidence interval and its interpretation.

Link with hypothesis tests.

Confidence limits for a Normal mean, with variance known.

Candidates will be expected to know how to apply the Normal distribution and use the standard error and obtain confidence intervals for the mean, rather than be concerned with any theoretical derivations.

Hypothesis tests for the mean of a Normal distribution with variance known.

Use of $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$.

Use of Central Limit theorem to extend hypothesis tests and confidence intervals to samples from non-Normal distributions. Use of large sample results to extend to the case in which the variance is unknown.

$\frac{\bar{X} - \mu}{S / \sqrt{n}}$ can be treated as $N(0, 1)$ when n is large. A knowledge of the t -distribution is not required.

Hypothesis test for the difference between the means of two Normal distributions with variances known.

Use of $\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0, 1)$.

Use of large sample results to extend to the case in which the population variances are unknown.

Use of $\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{S_x^2}{n_x} + \frac{S_y^2}{n_y}}} \sim N(0, 1)$.

A knowledge of the t -distribution is not required.

4. Goodness of fit and contingency tables

The null and alternative hypotheses. The use of $\sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$ as an approximate χ^2 statistic.

Degrees of freedom.

Applications to include the discrete uniform, binomial, Normal, Poisson and continuous uniform (rectangular) distributions. Lengthy calculations will not be required.

Candidates will be expected to determine the degrees of freedom when one or more parameters are estimated from the data. Cells should be combined when $E_i < 5$. Yates' correction is not required.

5. Regression and correlation

Spearman's rank correlation coefficient, its use, interpretation and limitations.

Testing the hypothesis that a correlation is zero.

Numerical questions involving ties will not be set. Some understanding of how to deal with ties will be expected.

Use of tables for Spearman's and product moment correlation coefficients.

Unit S4 – Statistics

The examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $\frac{1}{x}$, x^y , $\ln x$, e^x , sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Candidates will be expected to know and be able to recall and use the following formulae:

Type I error: $P(\text{reject } H_0 \mid H_0 \text{ true})$

Type II error: $P(\text{do not reject } H_0 \mid H_0 \text{ false})$

Prerequisites

A knowledge of the specification for S1, S2 and S3 and its prerequisites and associated formulae is assumed and may be tested.

SPECIFICATION

NOTES

1. Quality of tests and estimators

Type I and Type II errors.
Size and Power of Test.
The power test.

Simple applications. Calculation of the probability of a Type I or Type II error. Use of Type I and Type II errors and power function to indicate effectiveness of statistical tests. Questions will not be restricted to the Normal distribution.

Assessment of the quality of estimators.

Use of bias and the variance of an estimator to assess its quality. Consistent estimators.

2. One-sample procedures

Hypothesis test and confidence interval for the mean of a Normal distribution with unknown variance.

Use of $\frac{\bar{X} - \mu}{S / \sqrt{n}} \sim t_{n-1}$. Use of t -tables.

Hypothesis test and confidence interval for the variance of a Normal distribution.

Use of $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$. Use of χ^2 -tables.

3. Two-sample procedures

Hypothesis test that two independent random samples are from Normal populations with equal variances.

Use of $\frac{S_1^2}{S_2^2} \sim F_{n_1-1, n_2-1}$ under H_0 . Use of the tables of the F -distribution.

Use of the pooled estimate of variance.

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Hypothesis test and confidence interval for the difference between two means from independent Normal distributions when the variances are equal but unknown.

Use of t -distribution.

$$\frac{\bar{X} - \bar{Y}}{S \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}} \sim t_{n_x + n_y - 2}, \text{ under } H_0.$$

Paired t -test.

Unit D1 – Decision Mathematics

The examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $1/x$, x^y and memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Preamble

Candidates should be familiar with the terms defined in the glossary attached to this specification. Candidates should show clearly how an algorithm has been applied. Matrix representation will be required but matrix manipulation is not required. Candidates will be required to model and interpret situations, including cross-checking between models and reality.

SPECIFICATION

NOTES

1. Algorithms

The general ideas of algorithms.

The order of algorithm is not expected.

Implementation of an algorithm given by a flow chart or text.

Candidates should be familiar with

- (i) bin packing,
- (ii) bubble sort,
- (iii) quick sort,
- (iv) binary search.

When using the quick sort algorithm, the pivot should be chosen as the ‘number’ at the mid-point of the list.

2. Algorithms on graphs

The minimum spanning tree (minimum connector) problem. Prim’s and Kruskal’s (Greedy) algorithm.

Matrix representation for Prim’s algorithm is expected. Candidates will be expected to draw a network from a given matrix and also to write down the matrix associated with a network.

Dijkstra’s algorithm for finding the shortest path.

Planar and non-planar graphs. Planarity algorithm for graphs with a Hamiltonian cycle.

Candidates should know that K_5 and $K_{3,3}$ are non-planar. Kuratowski’s theorem is not required.

3. The route inspection problem

Algorithm for finding the shortest route around a network, travelling along every edge at least once and ending at the start vertex. The network will have up to four odd nodes.

Also known as the ‘Chinese postman’ problem. Candidates will be expected to consider all possible pairings of odd nodes. The application of Floyd’s algorithm to the odd nodes is not required.

4. Critical path analysis

Modelling of a project by an activity network, including the use of dummies.

A precedence table will be given. Activity on edge will be used.

Algorithm for finding the critical path. Earliest and latest event times. Earliest and latest start and finish times for activities. Total float. Gantt (cascade) charts. Scheduling.

5. Linear programming

Formulation of problems as linear programs.

Graphical solution of two variable problems using ruler and vertex methods. Consideration of problems where solutions must have integer values.

The Simplex algorithm and tableau for maximising problems.

Problems will be restricted to those with a maximum of three variables and three constraints, in addition to non-negativity conditions.

The use and meaning of slack variables.

6. Matchings

Use of bipartite graphs for modelling matchings. Complete matchings and maximal matchings.

Candidates will be required to use the maximum matching algorithm to improve a matching by finding alternating paths. No consideration of assignment is required.

Algorithm for obtaining a maximum matching.

7. Flows in networks

Algorithm for finding a maximum flow. Cuts and their capacity.

Vertex restrictions are not required. Only networks with directed edges will be considered. Only problems with upper capacities will be set.

Use of max flow – min cut theorem to verify that a flow is a maximum flow.

Multiple sources and sinks.

Glossary for D1

1. Algorithms

$[x]$ is the smallest integer which is greater than or equal to x , for example $[10.5] = 11$ and $[11] = 11$. In a list containing N names the 'middle' name has position $[\frac{1}{2}(N + 1)]$ so that if $N = 20$, this is $[10.5] = 11$ and if $N = 21$ it is $[11] = 11$.

2. Algorithms on graphs

A **graph** G consists of points (**vertices** or **nodes**) which are connected by lines (**edges** or **arcs**).

A **subgraph** of G is a graph, each of whose vertices belongs to G and each of whose edges belongs to G .

If a graph has a number associated with each edge (usually called its **weight**) then the graph is called a **weighted graph** or **network**.

The **degree** or **valency** of a vertex is the number of edges incident to it. A vertex is **odd (even)** if it has **odd (even)** degree.

A **path** is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, and in which no vertex appears more than once.

A **cycle (circuit)** is a closed path, ie the end vertex of the last edge is the start vertex of the first edge.

A cycle that passes through every vertex of a graph is called a **Hamiltonian cycle** and a graph in which a Hamiltonian cycle exists is said to be **Hamiltonian**.

Two vertices are **connected** if there is a path between them. A graph is **connected** if all its vertices are connected.

If the edges of a graph have a direction associated with them they are known as **directed edges** and the graph is known as a **digraph**.

A **tree** is a connected graph with no cycles.

A **spanning tree** of a graph G is a subgraph which includes all the vertices of G and is also a tree.

A **minimum spanning tree (MST)** is a spanning tree such that the total length of its edges is as small as possible. (This is sometimes called a **minimum connector**.)

A graph in which each of the n vertices is connected to every other vertex is called a **complete graph**. (The notation K_n will be used for such a graph with n vertices.)

A graph G is **planar** if it can be drawn in a plane in such a way that no two edges meet each other except at a vertex to which they are both incident.

4. Critical path analysis

The **total float** $F(i, j)$ of activity (i, j) is defined to be $F(i, j) = l_j - e_i - \text{duration}(i, j)$, where e_i is the earliest time for event i and l_j is the latest time for event j .

5. Linear programming

The **simplex tableau** for the linear programming problem:

$$\begin{aligned} &\text{Maximise } P = 14x + 12y + 13z, \\ &\text{Subject to } \quad 4x + 5y + 3z \leq 16, \\ &\quad \quad \quad 5x + 4y + 6z \leq 24, \end{aligned}$$

will be written as

Basic variable	x	y	z	r	s	Value
r	4	5	3	1	0	16
s	5	4	6	0	1	24
P	-14	-12	-13	0	0	0

where r and s are slack variables.

6. Matchings

A **bipartite graph** consists of two sets of vertices X and Y . The edges only join vertices in X to vertices in Y , not vertices within a set. (If there are r vertices in X and s vertices in Y then this graph is $K_{r,s}$.)

A **matching** is the pairing of some or all of the elements of one set, X , with elements of the second set, Y . If every member of X is paired to a member of Y the matching is said to be a **complete matching**.

7. Flows in networks

A **cut**, in a network with source S and sink T , is a set of arcs (edges) whose removal separates the network into two parts X and Y , where X contains at least S and Y contains at least T . The **capacity of a cut** is the sum of the capacities of those arcs in the cut which are directed from X to Y .

If a network has several sources S_1, S_2, \dots , then these can be connected to a single **supersource** S . The capacity of the edge joining S to S_1 is the sum of the capacities of the edges leaving S_1 .

If a network has several sinks T_1, T_2, \dots , then these can be connected to a **supersink** T . The capacity of the edge joining T_1 to T is the sum of the capacities of the edges entering T_1 .

Unit D2 – Decision Mathematics

The examination

The examination will consist of one 1½ hour paper. The paper will contain about seven questions with varying mark allocations per question which will be stated on the paper. All questions may be attempted.

Candidates are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Candidates are expected to have available a calculator with at least the following keys: +, −, ×, ÷, π , x^2 , \sqrt{x} , $1/x$, x^y and memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Prerequisites

Candidates should be familiar with the material in, and the glossary attached to, the D1 specification.

Preamble

Candidates should be familiar with the terms defined in the glossary attached to this specification. Candidates should show clearly how an algorithm has been applied.

SPECIFICATION

NOTES

1. Transportation problems

N	The north-west corner method for finding an initial basic feasible solution.	Problems will be restricted to three sources and three destinations. To include the idea of a dummy location. The idea of degeneracy will be required.
S	Use of the stepping-stone method for obtaining an improved solution. Improvement coefficients.	Candidates should state a specific entering cell and a specific exiting cell.
S	Formulation as a linear programming problem.	

2. Allocation (assignment) problems

N	Reduction of cost matrix.	The size of the cost matrix will be at most 4×4 .
S	Use of the Hungarian algorithm to find least cost allocation.	To include the idea of a dummy location.
S	Modification of method to deal with a maximum profit allocation. Formulation as a linear programming problem.	

3. The travelling salesman problem

- N The practical and classical problems. The classical problem for complete graphs satisfying the triangle inequality.
- S Determination of upper and lower bounds using minimum spanning tree methods. Including the use of short cuts to improve upper bound.
- S The nearest neighbour algorithm. Including the conversion of a network into a complete network of shortest 'distances'.

4. Game theory

- N Two person zero-sum games and the pay-off matrix. Identification of play safe strategies and stable solutions (saddle points).
- N Reduction of pay-off matrix using dominance arguments. Optimal mixed strategies for a game with no stable solution. Use of graphical methods for $2 \times n$ or $n \times 2$ games where $n = 1, 2$ or 3 .
- S Conversion of 3×2 and 3×3 games to linear programming problems.
- S Solution of the resulting linear programming problems for the 3×2 and 3×3 game.

5. Dynamic programming

- N Principles of dynamic programming. Bellman's principle of optimality.
- S Stage and State variables. Use of tabulation to solve problems involving finding a maximum, minimum, minimax or maximin. Problems may be set using a network or table. For network problems, the network will be given.

Glossary for D2

1. Transportation problems

In the **north-west corner method** the upper left-hand cell is considered first and as many units as possible sent by this route.

The **stepping stone method** is an iterative procedure for moving from an initial feasible solution to an optimal solution.

Degeneracy occurs in a transportation problem, with m rows and n columns, when the number of occupied cells is less than $(m + n - 1)$.

In the transportation problem:

The **shadow costs** R_i , for the i th row, and K_j , for the j th column, are obtained by solving $R_i + K_j = C_{ij}$ for **occupied cells**, taking $R_1 = 0$ arbitrarily.

The **improvement index** I_{ij} for an **unoccupied cell** is defined by $I_{ij} = C_{ij} - R_i - K_j$.

3. The travelling salesman problem

The **travelling salesman problem** is ‘find a route of minimum length which visits every vertex in an undirected network’. In the ‘**classical**’ problem, each vertex is visited once only. In the ‘**practical**’ problem, a vertex may be revisited.

For three vertices A , B and C the **triangular inequality** is ‘length $AB \leq$ length $AC +$ length CB ’.

A **walk** in a network is a finite sequence of edges such that the end vertex of one edge is the start vertex of the next.

A walk which visits every vertex, returning to its starting vertex, is called a **tour**.

4. Game theory

A **two-person game** is one in which only two parties can play.

A **zero-sum** game is one in which the sum of the losses for one player is equal to the sum of the gains for the other player.

5. Dynamic programming

Bellman’s principle for dynamic programming is ‘Any part of an optimal path is optimal.’

The **min-max route** is the one in which the maximum length of the arcs used is as small as possible.

The **max-min route** is the one in which the minimum length of the arcs used is as large as possible.

Grade descriptions

The following grade descriptions indicate the level of attainment characteristic of grades A, C and E at Advanced GCE. They give a general indication of the required learning outcomes at the specified grades. The descriptions should be interpreted in relation to the content outlined in the specification; they are not designed to define that content. The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

Grade A

Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs. When confronted with unstructured problems they can often devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are sometimes noticed and corrected.

Candidates recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They correctly refer results from calculations using the model to the original situation; they give sensible interpretations of their results in the context of the original realistic situation. They make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts. They correctly refer the results of calculations back to the given context and usually make sensible comments or predictions. They can distil the essential mathematical information from extended pieces of prose having mathematical content. They can comment meaningfully on the mathematical information.

Candidates make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.

Grade C

Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and sometimes proceed logically through extended arguments or proofs. When confronted with unstructured problems they sometimes devise and implement an effective and efficient solution strategy. They occasionally notice and correct errors in their calculations.

Candidates recall or recognise most of the standard models that are needed and usually select appropriate ones to represent a variety of situations in the real world. They often correctly refer results from calculations using the model to the original situation; they sometimes give sensible interpretations of their results in the context of the original realistic situation. They sometimes make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of most translations into mathematics of common realistic contexts. They often correctly refer the results of calculations back to the given context and sometimes make sensible comments or predictions. They distil much of the essential mathematical information from extended pieces of prose having mathematical content. They give some useful comments on this mathematical information.

Candidates usually make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are sometimes aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

Grade E

Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.

Candidates recall or recognise some of the standard models that are needed and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes correctly refer results from calculations using the model to the original situation; they try to interpret their results in the context of the original realistic situation.

Candidates sometimes comprehend or understand the meaning of translations in mathematics of common realistic contexts. They sometimes correctly refer the results of calculations back to the given context and attempt to give comments or predictions. They distil some of the essential mathematical information from extended pieces of prose having mathematical content. They attempt to comment on this mathematical information.

Candidates often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They often present results to an appropriate degree of accuracy.

Textbooks and other resources

Textbooks

New editions of textbooks for AS/Advanced GCE Mathematics will be available for the start of courses leading to examinations covered by this specification. These revised editions will support the new Edexcel AS/Advanced GCE specification and will provide:

coverage of the general topic areas of areas of the specification

progression from GCSE, through AS and up to Advanced GCE

examination-related exercises

examination practice.

Support and training

Training

Each year Edexcel provides a programme of training courses covering aspects of the specifications and assessment. These courses take place throughout the country. For further information on what is planned, please consult the annual Training and Professional Development Guide, which is sent to all centres, or contact:

Professional Development and Training
Edexcel Foundation
Stewart House
32 Russell Square
London WC1B 5DN

Tel: 0870 240 9800
Fax: 020 7758 5951
E-mail: trainingenquiries@edexcel.org.uk

Mark schemes with examiners' comments

Mark schemes and examiners' comments will be issued to centres for mathematics after each examination series.

Additional copies may be obtained from Edexcel Publications.

Support materials

The following specification support materials will be available to centres:

Specimen papers

Other materials will be made available to centres during the lifetime of the specification in response to centres' needs.

Additional copies of support material may be obtained from:

Edexcel Publications
Adamsway
Mansfield
Notts NG18 4LN

Tel: 01623 467467
Fax: 01623 450481
E-mail: publications@maillin.co.uk

Regional offices

Further advice and guidance is available through a national network of regional offices. For details of your nearest office please call the Edexcel Customer Response Centre on 0870 240 9800.

Appendices

Key Skills development

The AS/Advanced GCE in Mathematics offers a range of opportunities for candidates to both develop their Key Skills, in

- Application of Number
- Communication
- Information Technology
- Improving own Learning and Performance
- Working with Others
- Problem Solving

and generate assessed evidence for their portfolios.

Copies of the Key Skills specifications can be ordered through our publications catalogue. The individual Key Skills units are divided into three parts:

- Part A: what you need to know – this identifies the underpinning knowledge and skills required
- Part B: what you must do – this identifies the evidence that candidates must produce for their portfolios
- Part C: guidance – this gives examples of possible activities and types of evidence that may be generated.

This Advanced GCE specification signposts development and internal assessment opportunities which are based on Part B of the level 3 Key Skills units.

Additional guidance is available for those candidates working towards levels 2 or 4 for any of the individual Key Skills units.

The evidence generated through this Advanced GCE can be internally assessed and contribute to the candidate's Key Skills portfolio.

Each unit within the Advanced GCE in Mathematics can provide opportunities for the development of all six of the Key Skills identified. This section identifies the Key Skills evidence requirements and also provides a mapping of those opportunities. Candidates will need to have opportunities to develop their skills over time before they are ready for assessment. For each skill you will find illustrative activities that will aid this Key Skill development and facilitate the generation of appropriate portfolio evidence. To assist in the recording of Key Skills evidence Edexcel has produced recording documentation which can be ordered from our publications catalogue.

Appendix 1: Mapping of Key Skills – summary table

Key Skill	C1	C2	C3	C4	FP1	FP2	FP3	M1	M2	M3	M4	M5	S1	S2	S3	S4	D1	D2
Application of Number																		
N3.1	X	X	X					X	X	X			X	X	X	X		
N3.2	X	X	X					X	X	X			X	X	X	X		
N3.3	X	X	X					X	X	X			X	X	X	X		
Information Technology																		
IT3.1													X	X	X	X		
IT3.2													X	X	X	X		
IT3.3													X	X	X	X		
Communication																		
C3.1a													X	X	X	X		
C3.1b													X	X	X	X		
C3.2															X			
C3.3															X			
Working with Others																		
WO3.1													X	X	X	X		
WO3.2													X	X	X	X		
WO3.3													X	X	X	X		
Improving own Performance																		
LC3.1								X	X	X	X	X	X	X	X	X		
LC3.2								X	X	X	X	X	X	X	X	X		
LC3.3								X	X	X	X	X	X	X	X	X		
Problem Solving																		
PS3.1	X	X	X		X	X	X	X	X	X	X	X	X	X	X	X		
PS3.2	X	X	X		X	X	X	X	X	X	X	X	X	X	X	X		
PS3.3	X	X	X		X	X	X	X	X	X	X	X	X	X	X	X		
PS3.4	X	X	X		X	X	X	X	X	X	X	X	X	X	X	X		

Key Skills development opportunities

Application of Number Level 3

The AS/Advanced units in Core and Further Pure Mathematics provide opportunities for candidates both to develop the Key Skill of Application of Number and also to generate evidence for their portfolio. As well as undertaking tasks related to the three areas of evidence required, candidates are also required to undertake a substantial and complex activity. This will involve candidates planning, obtaining and interpreting information, using this information when carrying out multi-stage calculations and explaining how the results of the calculations meet the purpose of the activity and justifying their methods.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment
N3.1	Plan and interpret information from two different sources, including a large data set.	<p>Candidates are required to:</p> <ul style="list-style-type: none"> • plan how to obtain and use the information required to meet the purpose of their activity • obtain the relevant information; and • choose appropriate methods for obtaining the results they need and justify their choice. <p>For example, the criteria for N3.1 are partially satisfied when:</p> <ul style="list-style-type: none"> • choosing an appropriate degree of accuracy to work to in calculations • using indices • using compound measures in calculus • setting up a set of logical sequences whilst working towards a proof • using large and small numbers in exponential growth and decay • setting up appropriate differential equations to model a real-life situation • set up a mathematical model of a practical situation. <p>NB Candidates should be given topics where they have to choose the method of calculation.</p> <p>In completing the Core and Further Pure Mathematics units candidates would not necessarily:</p> <ul style="list-style-type: none"> • plan a substantial and complex activity by breaking it down into a series of tasks • obtain relevant information from different sources including a large data set (over 50 items) • make accurate and reliable observations over time and use suitable equipment to measure in a variety of appropriate units.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment
N3.2	<p>Carry out multi-stage calculations to do with:</p> <ul style="list-style-type: none"> a amounts and sizes b scales and proportion c handling statistics d rearranging and handling formulae <p>They should work with a large data set on at least one occasion</p>	<p>Candidates must:</p> <ul style="list-style-type: none"> • carry out their calculations to appropriate levels of accuracy, clearly showing their methods; and • check methods and results to help ensure errors are found and corrected. <p>For example, the criteria for N3.2 are partially satisfied when:</p> <ul style="list-style-type: none"> • candidates perform multi-stage calculations throughout the units, particularly when using calculus in real-life contexts eg functions, integration by substitution and by parts, product, quotient and chain rule, differential equations, etc • they choose an appropriate level of accuracy to work to • they calculate and use angles when using trigonometry • they perform calculations involving rate of change, particularly in practical contexts • they form, rearrange and use formulae, equations and expressions • they check their work using alternative methods • use vectors to calculate lengths and angles in practical situations • perform the appropriate calculations in a mathematical model of a real-life situation. <p>In completing the Core and Further Pure Mathematics units candidates would not necessarily:</p> <ul style="list-style-type: none"> • work with a large data set (over 50 items), using measures of average and range to compare distributions and estimate mean, median and range of grouped data.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment
N3.3	Interpret results of their calculations, present their findings and justify their methods. They must use at least one graph, one chart and one diagram.	<p>On the basis of their findings, candidates must:</p> <ul style="list-style-type: none"> • select appropriate methods of presentation and justify their choice • present their findings effectively; and • explain how the results of their calculations relate to the purpose of their activity. <p>For example, the criteria for N3.3 are partially satisfied when:</p> <ul style="list-style-type: none"> • methods of calculation are discussed and justified. Candidates would benefit from having a choice of possible methods, eg numerical against calculus methods, and asked to choose and justify their choice • efforts are made to consider the models of real-life situations used, with the limitations of the model realised and discussed. Constraints on methods and calculations used should be applied, eg numerically estimating the area under a curve as opposed to calculating through integration • using calculus methods in practical situations to arrive at an optimum choice with justification • candidates discuss the benefits of a mathematical technique, eg using vector notation and scalar products to calculate angles in a 3D practical context. <p>In completing the Core and Further Pure Mathematics units candidates would not necessarily:</p> <ul style="list-style-type: none"> • select and use appropriate methods to illustrate findings, show trends and make comparisons • draw appropriate conclusions based on their findings, including how possible sources of error might have affected their results.

Evidence

Candidate evidence for application of number could include:

- a description of the substantial and complex activity and tasks. A plan for obtaining and using the information required
- copies of source material, including a note of the large data set and, if applicable, a statement from someone who has checked the accuracy of any measurements or observations
- records of the information they obtained. A justification of methods selected for achieving the required results
- records of the calculations showing methods used and levels of accuracy
- notes of the large data set used and how they checked methods and results
- report of their findings, including justification of their presentation methods and explanations of how their results relate to their activity. At least one chart, one graph and one diagram.

Application of Number level 3

The AS/Advanced units in mechanics provide opportunities for candidates both to develop the Key Skill of Application of Number and also to generate evidence for their portfolio. As well as undertaking tasks related to the three areas of evidence required candidates are also required, to undertake a substantial and complex activity. This will involve candidates planning, obtaining and interpreting information, using this information when carrying out multi-stage calculations and explaining how the results of the calculations meet the purpose of the activity and justifying their methods.

Key skill portfolio evidence requirement		Opportunities for development or internal assessment
N3.1	Plan, and interpret information from two different sources, including a large data set.	<p>Candidates are required to:</p> <ul style="list-style-type: none"> • plan how to obtain and use the information required to meet the purpose of their activity • obtain the relevant information; and • choose appropriate methods for obtaining the results they need and justify their choice. <p>For example, the criteria for N3.1 are partially satisfied when:</p> <ul style="list-style-type: none"> • setting up a mathematical model of a real-life situation, particularly if all sections of the modelling cycle are planned, executed and analysed by the candidate • choosing an appropriate degree of accuracy to work to in calculations • setting up experiments that require the use of suitable equipment, observations and the recording of results • using standard form and indices to represent large and small numbers • using compound measures of displacement, velocity and acceleration in addition to those involved in calculus. <p>In completing the mechanics units candidates would not necessarily:</p> <ul style="list-style-type: none"> • obtain relevant information from different sources including a large data set (over 50 items).

Key Skill portfolio evidence requirement	Opportunities for development or internal assessment
<p>N3.2 Carry out multi-stage calculations to do with:</p> <ul style="list-style-type: none"> a amounts and sizes b scales and proportion c handling statistics d rearranging and handling formulae. <p>They should work a large data set on at least one occasion</p>	<p>Candidates must:</p> <ul style="list-style-type: none"> • carry out their calculations to appropriate levels of accuracy, clearly showing their methods; and • check methods and results to help ensure errors are found and corrected. <p>For example, the criteria for N3.2 are partially satisfied when:</p> <ul style="list-style-type: none"> • candidates perform multi-stage calculations throughout the units, particularly when using calculus in real-life contexts – for example, resolving forces through the use of vectors and their components, velocity and acceleration when displacement is a function of time, centres of mass of uniform plane figures, differential equations, etc • they choose an appropriate level of accuracy to work to • they perform calculations involving rate of change, particularly in practical contexts • they form, rearrange and use formulae, equations and expressions • they check their work using alternative methods • they use vectors to calculate lengths and angles in practical situations • they perform the appropriate calculations in a mathematical model of a real-life situation • they take measurements from scale drawings, eg to model forces, etc. <p>In completing the mechanics units candidates would not necessarily: work with large data sets (over 50 items) using measures of average and range to compare distributions, and estimate mean, median and range of grouped data.</p>

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment
N3.3	Interpret results of their calculations, present their findings and justify their methods. They must use at least one graph, one chart and one diagram.	<p>On the basis of their findings, candidates must:</p> <ul style="list-style-type: none"> • select appropriate methods of presentation and justify their choice • present their findings effectively; and • explain how the results of their calculations relate to the purpose of their activity. <p>For example, the criteria for N3.3 are partially satisfied when:</p> <ul style="list-style-type: none"> • methods of calculation are discussed and justified. candidates would benefit from having a choice of possible methods, eg numerical as against graphical or calculus methods, and are asked to choose and justify their choice • efforts are made to consider the models of real-life situations used, with the limitations of the model realised and discussed. Constraints on methods and calculations used should be applied, eg numerically estimating the area under a curve as opposed to calculating through integration • using calculus methods in practical situations to arrive at an optimum choice with justification • candidates discuss the benefits of a mathematical technique, eg using vector notation and scalar products to calculate angles in a 3-D practical context. <p>In completing the Core and Further Pure Mathematics units candidates would not necessarily: select and use appropriate methods to illustrate findings, show trends and make comparisons.</p>

Evidence

Candidate evidence for application of number could include:

- a description of the substantial and complex activity and tasks. A plan for obtaining and using the information required
- copies of source material, including a note of the large data set and, if applicable, a statement from someone who has checked the accuracy of any measurements or observations
- records of the information they obtained. A justification of methods selected for achieving the required results
- records of the calculations showing methods used and levels of accuracy
- notes of the large data set used and how they checked methods and results
- report of their findings, including justification of their presentation methods and explanations of how their results relate to their activity. At least one chart, one graph and one diagram.

Application of Number level 3

The AS/Advanced units in statistics provide opportunities for candidates both to develop the Key Skill of Application of Number and also to generate evidence for their portfolio. As well as undertaking tasks related to the three areas of evidence required candidates are also required to undertake a substantial and complex activity. This will involve candidates planning, obtaining and interpreting information, using this information when carrying out multi-stage calculations and explaining how the results of the calculations meet the purpose of the activity and justifying their methods.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment
N3.1	Plan, and interpret information from two different sources, including a large data set.	<p>Candidates are required to:</p> <ul style="list-style-type: none"> • plan how to obtain and use the information required to meet the purpose of their activity • obtain the relevant information; and • choose appropriate methods for obtaining the results they need and justify their choice. <p>For example, the criteria for N3.1 are fully satisfied when:</p> <ul style="list-style-type: none"> • choosing an appropriate degree of accuracy to work to in calculations • obtaining information from a large data set (at least 50 items) • making accurate and reliable observations over time, eg carrying out a survey of supermarket prices over a six month period • planning a substantial and complex activity when preparing their coursework • using very large and small numbers as part of their calculations • setting up a mathematical model of a practical situation • choosing the appropriate statistical technique to apply in a given situation. <p>NB Candidates should be given topics where they have to choose the method of calculation.</p> <p>In completing the statistics units candidates would satisfy all the criteria for N3.1.</p>

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment
N3.2	<p>Carry out multi-stage calculations to do with:</p> <ul style="list-style-type: none"> a amounts and sizes b scales and proportion c handling statistics d rearranging and handling formulae <p>They should work with a large data set on at least one occasion</p>	<p>Candidates must:</p> <ul style="list-style-type: none"> • carry out their calculations to appropriate levels of accuracy, clearly showing their methods; and • check methods and results to help ensure errors are found and corrected. <p>For example the criteria for N3.2 are partially satisfied when:</p> <ul style="list-style-type: none"> • candidates perform multi-stage calculations throughout the units, particularly when using Normal, binomial and Poisson distributions in real-life contexts • they use an appropriate level of accuracy • they use powers and roots in calculations, eg using the formula for standard deviation • they form, rearrange and use formulae, equations and expressions, eg finding, using and applying z values to a normal distribution • using proportion in the context of probability • they check their work using alternative methods, eg comparing algebraic, graphical and numerical techniques for calculating the median of a grouped frequency distribution • performing the appropriate calculations in a mathematical model of a real-life situation. <p>In completing the statistics units candidates would not necessarily:</p> <ul style="list-style-type: none"> • work with missing angles and sides in a right-angled triangle from known sides and angles • work out proportional change • work out measurements from scale drawings.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment
N3.3	Interpret results of their calculations, present their findings and justify their methods. They must use at least one graph, one chart and one diagram.	<p>On the basis of their findings, candidates must:</p> <ul style="list-style-type: none"> • select appropriate methods of presentation and justify their choice • present their findings effectively; and • explain how the results of their calculations relate to the purpose of their activity. <p>For example, the criteria for N3.3 are fully satisfied when:</p> <ul style="list-style-type: none"> • methods of calculation are discussed and justified. Candidates would benefit from having a choice of possible methods, eg numerical against graphical methods, and being asked to choose and justify their choice • efforts are made to consider the models of real-life situations used, with the limitations of the model realised and discussed. Constraints on methods and calculations used should be applied, eg using the mid-value to calculate an estimate of the mean of a grouped frequency distribution • candidates discuss the benefits of a mathematical technique, eg comparing measures of central tendency in different situations • selecting appropriate methods to illustrate findings, show trends and make comparisons eg regression and correlation, significance testing, chi-squared testing, etc • constructing and labelling charts, diagrams..., using accepted conventions • drawing appropriate conclusions based on candidates' findings, including how possible sources of error might have affected their results, eg confidence intervals, significance testing, etc • explaining how their results relate to the purpose of their activity, eg hypothesis testing. <p>In completing the statistics units candidates would satisfy all the criteria for N3.3 except</p> <ul style="list-style-type: none"> • construct and label ...scale drawings.

Evidence

Candidate evidence for application of number could include:

- a description of the substantial and complex activity and tasks. A plan for obtaining and using the information required
- copies of source material, including a note of the large data set and, if applicable, a statement from someone who has checked the accuracy of any measurements or observations
- records of the information they obtained. A justification of methods selected for achieving the required results
- records of the calculations showing methods used and levels of accuracy
- notes of the large data set used and how they checked methods and results
- report of their findings, including justification of their presentation methods and explanations of how their results relate to their activity. At least one chart, one graph and one diagram.

Information Technology level 3

When producing work for their Advanced GCE in Mathematics, candidates will have numerous opportunities to use information technology. The Internet, CD ROMs, etc could be used to collect information, particularly when collecting data for use in the statistics units S1, S2 and S3. Documents can be produced using relevant software and images may be incorporated in those documents. Early drafts of documents could be E-mailed to tutors for initial comments and feedback.

For this Key Skill, candidates are required to carry out at least one ‘substantial activity’. This is defined as ‘an activity that includes a number of related tasks, where the results of one task will affect the carrying out of the others’. The activity should generate evidence for all three areas of evidence required in Part B of the IT unit. If candidates undertaking coursework as part of their AS/A2 units in statistics use information technology, they will have opportunities to generate evidence for all three sections identified as part of a ‘substantial activity’.

Mathematics candidates should utilise IT as a modelling tool, particularly when using graphical calculators and spreadsheets. Accounts of their use in this way should be encouraged as part of candidates’ IT portfolio.

In addition, candidates will be able to use information technology to generate evidence for the Communication Key Skill. For example, the extended document with images, required for C3.3, could be generated using appropriate software.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment	
IT3.1	Plan, and use different sources to search for, and select, information required for two different purposes	S1 – 4	<p>Candidates will need to plan, and document, how they are to use IT as part of the activity, including how they will search for and incorporate relevant information from different electronic sources. These may include the Internet and CD ROM. Information selected must be relevant and of the appropriate quality.</p> <p>For example, opportunities for partially satisfying this criteria include:</p> <ul style="list-style-type: none"> • collecting data in secondary form from a variety of Internet sources including: <ol style="list-style-type: none"> (i) the DfEE website for educational performance tables (ii) the Office of Health Economics, etc.
IT3.2	Explore, develop and exchange information and derive new information to meet two different purposes	S1 – 4	<p>Candidates are required to bring together, in a consistent format, their selected information and use automated routines as appropriate.</p> <p>For example, opportunities for partially satisfying this criterion include:</p> <ul style="list-style-type: none"> • interrogating a professional database to extract relevant data for use in S1,S2 and S3 • recording data from an experiment in a spreadsheet and using its functions to calculate appropriate statistics and/or values, eg modelling a tide table to a cosine curve and calculating the error bounds of the model v actual data in C2 and C3.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment	
IT3.3	<p>Present information from different sources for two different purposes and audiences.</p> <p>This work must include at least one example of text, one example of images and one example of numbers</p>	S1 – 4	<p>Candidates need to:</p> <ul style="list-style-type: none"> develop a structure, which may involve the modification of templates, the application of page numbers, dates, etc. <p>Tutors may provide early feedback on layout on content and style that will result in formatting changes (early drafts should be kept as portfolio evidence).</p> <p>The final format should be suitable for its purpose and audience, eg AS coursework, ohts/handouts for a presentation, etc. The document should have accurate spelling (use of spell-checker) and have been proof-read.</p>

Evidence

Candidate evidence for information technology could include:

- tutor observation records
- preparatory plans
- printouts with annotations
- draft documents.

Communication level 3

For the Communication Key Skill, candidates are required to hold discussions and give presentations, read and synthesise information and write documents. Candidates will be able to develop all of these skills through an appropriate teaching and learning programme based on this Advanced GCE.

The Statistics units offer the most obvious opportunities for gathering evidence and developing this Key Skill.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment	
C3.1a	Contribute to a group discussion about a complex subject	S1 – 4	<p>Many of the topics in this specification are suitable as the basis of a group discussion. The discussion must be about a complex subject. This may be based on a number of ideas, some of which may be abstract, very detailed and/or sensitive. Specialist vocabulary may be used in the discussion. During the discussion candidates should make clear and relevant contributions, and develop points and ideas whilst listening and responding sensitively to others. They should also create opportunities for others to contribute as appropriate.</p> <p>Many topics in the statistics specifications lend themselves to group discussion, eg the validity of one method of significance testing or sampling against another for particular populations.</p>
C3.1b	Make a presentation about a complex subject, using at least one image to illustrate complex points	S1 – 4	<p>Following a period of research candidates could be given the opportunity to present their findings to the rest of the group.</p> <p>During the presentation candidates should speak clearly and use a style that is appropriate to their audience and the subject. The presentation should have a logical structure that allows the audience to follow the sequence of information and ideas. The presentation should include an appropriate range of techniques such as:</p> <p>the use of examples to illustrate complex points, audience experience used to involve the audience, tone of voice varied, etc.</p> <p>Where appropriate, images should be used both to illustrate points and to help engage the audience. Images could include charts and diagrams or other statistical diagrams. At least one image should be used to illustrate and help convey a complex point.</p> <p>Candidates could also make a presentation relating to topics in the specification, eg forming hypotheses and demonstrating how and why they accepted or rejected them.</p>

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment	
C3.2	<p>Read and synthesise information from two extended documents about a complex subject</p> <p>One of these documents should include at least one image</p>	S3	<p>Candidates will have a number of opportunities to read and synthesise information from two extended documents. For example, as part of their preparation for the discussion and presentation of a complex subject, candidates will need to carry out preliminary research. Also, as candidates undertake research for their coursework they will need to refer to and synthesise information from a variety of sources.</p> <p>Extended documents may include textbooks and reports and articles of more than three pages. At least one of these documents should contain an image from which candidates can draw appropriate and relevant information.</p> <p>Candidates will need to select and read material that contains relevant information. From this information they will need to identify accurately and compare the lines of reasoning and main points from the text and images. Candidates will then need to synthesise this information into a relevant form, eg for a presentation, discussion or an essay.</p> <p>Candidates may have to do some background reading for S1—S3.</p>
C3.3	<p>Write two different types of documents about complex subjects</p> <p>One piece of writing should be an extended document and include at least one image</p>	S3	<p>Candidates are required to produce two different types of document. At least one of these should be an extended document, for example a report or an essay of more than three pages.</p> <p>The document should have a form and style of writing which are fit both for its purpose and the complex subject matter covered. At least one of the documents should include an appropriate image that contains and effectively conveys relevant information. Specialist vocabulary should be used where appropriate and the information in the document should be clearly and coherently organised eg through the use of headings, paragraphs, etc.</p> <p>Candidates should ensure that the text is legible and that spelling, punctuation and grammar are accurate.</p>

Evidence

Candidate evidence for communication could include:

- tutor observation records
- preparatory notes
- audio/video tapes
- notes based on documents read
- essays.

Working with Others level 3

To achieve this Key Skill, candidates are required to carry out at least two complex activities. Candidates will negotiate the overall objective of the activity with others and plan a course of action. Initially the component tasks of the activity, and their relationships, may not be immediately clear. Within the activity, the topics covered may include ideas that may be some or all of the following: detailed, abstract, unfamiliar, sensitive.

During the activity the candidate must work in both group-based and one-to-one situations.

Few of the mathematics units lend themselves readily to ‘working with others’ as this often relies on a project/group work approach to the delivery of the content. The statistics units can be modified to incorporate a more collaborative approach throughout, with the coursework component of S3 an area for further opportunities to satisfy WO 3.1, 3.2 and 3.3.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment	
WO3.1	Plan the activity with others, agreeing objectives, responsibilities and working arrangements	S1 – 4	<p>Candidates could work in groups of six to eight and be required to investigate a given topic. Initial work will require identification of and agreeing of objectives and planning how to meet these, including any necessary action and resources required. The group needs to agree responsibilities and working arrangements.</p> <p>For example, throughout the data collection activities in all statistics units, candidates should be encouraged to:</p> <ul style="list-style-type: none"> • discuss and agree on hypotheses to be tested • share out the data collection within the group, taking the opportunity to discuss relevant sampling techniques • effectively manage the time of each group member, agreeing targets and deadlines. <p>Partial satisfaction of these criteria relies on the teacher creating opportunities for data collection rather than allocating data that has already been prepared.</p>
WO3.2	Work towards achieving the agreed objectives, seeking to establish and maintain co-operative working relationships in meeting your responsibilities	S1 – 4	<p>For example, throughout the data collection activities in all statistics units, candidates should be encouraged to:</p> <ul style="list-style-type: none"> • discuss and agree on hypotheses to be tested • share out the data collection within the group, taking the opportunity to discuss relevant sampling techniques • effectively manage the time of each group member, agreeing targets and deadlines.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment	
WO3.3	Review the activity with others against the agreed objectives and agree ways of enhancing collaborative work	S1 – 4	For example, throughout the data collection activities in all statistics units, candidates should be encouraged to: <ul style="list-style-type: none"> • review outcomes against the agreed hypotheses • identify factors that have influenced the outcome • agree on the ways in which the activity could have been carried out more effectively.

Evidence

Candidate evidence for working with others could include:

- tutor observation records
- preparatory plans
- records of process and progress made
- evaluative reports.

Improving own Learning and Performance level 3

Within Advanced GCE in Mathematics, candidates will have opportunities to develop and generate evidence, which meets part of the evidence requirement of this Key Skill.

To achieve this Key Skill candidates will need to carry out two study-based learning activities and two activity-based learning activities. Most of the units forming the Advanced GCE in Mathematics will provide opportunities for candidates to undertake study-based learning. Evidence for activity-based learning will depend on the approach adopted by the teacher. Fieldwork within the mechanics and statistics units readily lends itself to satisfying some of the criteria for LC3.1, LC3.2 and LC3.3.

One of the study-based learning activities must contain at least one complex task and periods of self-directed learning. Activities that generate evidence for this skill should take place over an extended period of time, eg three months. Over the period of the activity candidates should seek and receive feedback, from tutors and others, on their target setting and performance.

Any substantial project work (including coursework) comprises suitable study-based learning activities and may be used to generate evidence for this Key Skill.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment	
LC3.1	Agree targets and plan how these will be met, using support from appropriate others	M1 – 5 S1 – 4	<p>Candidates plan how they are to produce their coursework. This will include setting realistic dates and targets and identification of potential problems and alternative courses of action. This will be determined with advice from others, eg their tutor.</p> <p>In mathematics the candidate could:</p> <ul style="list-style-type: none"> plan a rigorous timetable for home study, reviews and tutorials for all the units
LC3.2	Use your plan, seeking feedback and support from relevant sources to help meet your targets, and use different ways of learning to meet new demands	M1 – 5 S1 – 4	<p>Candidates use the plan effectively when producing:</p> <ul style="list-style-type: none"> an extended account of a mechanics experiment/model. <p>This will involve:</p> <ul style="list-style-type: none"> prioritising action managing their time effectively; and revising their plan as necessary. <p>The candidate should:</p> <ul style="list-style-type: none"> seek and use feedback and support and draw on different approaches to learning as outlined in their detailed plan of action.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment	
LC3.3	Review progress establishing evidence of achievements, and agree action for improving performance	M1 – 5 S1 – 4	Candidates should review their own progress and the quality of their learning and performance. They should identify targets met, providing evidence of achievements from relevant sources. They should identify with others, eg their tutor, action for improving their performance.

Evidence

Candidate evidence for improving own learning and performance could include:

- tutor records
- annotated action plans
- records of discussions
- learning log
- work produced.

Problem Solving level 3

For this Key Skill candidates are required to apply their problem solving skills to complex activities. They need to show that they can recognise, explore and describe problems, generate ways of solving problems, implement options and check whether the problem has been solved.

Key Skill portfolio evidence requirement		Opportunities for development or internal assessment	
PS3.1	Recognise, explore and describe the problem, and agree the standards for its solution	C1 – 4 M1 – 5 S1 – 4	Candidates will need to identify the problem and explore its main features and agree standards that have to be met to show successful resolution of the problem. For example, discuss and agree common methods of presentation throughout the mathematics units, including: <ul style="list-style-type: none"> • consistent and accurate use of standard units of length, mass and capacity in addition to compound units in C1, C2 and C3, M1, M2 and M3 • conventional notation in calculus, vector geometry, etc • formal methods of proof in C1 and C3, etc.
PS3.2	Generate and compare at least two options which could be used to solve the problem, and justify the option for taking forward	C1 – 4 M1 – 5 S1 – 4	Candidates are required to select and use appropriate methods for generating different options for tackling the problem and compare the features of each option, selecting the most suitable one. Candidates need to be aware that there may be more than one method of solution of mathematical problems and be capable of comparing the effectiveness of each; comparing numerical methods to calculus methods for calculating the area under a curve in C1 and C2; comparing the effectiveness of grouping data when estimating the mean in S1; comparing various forms of statistical diagrams for presenting data; etc.
PS3.3	Plan and implement at least one option for solving the problem, and review progress towards its solution	C1 – 4 M1 – 5 S1 – 4	The implementation of the chosen option will need to be planned and permission gained to implement it. Implementation of the plan should involve full use of support and feedback from others with progress reviews and alterations to the plan as necessary.
PS3.4	Agree and apply methods to check whether the problem has been solved, describe the results and review the approach taken.	C1 – 4 M1 – 5 S1 – 4	On completion the outcomes need to be checked against the standards agreed at the start. The results of this should be recorded and the approach taken reviewed. For example: One complete run through a modelling cycle in mechanics, reviewing the model's effectiveness and redesigning the process to take account of any changes.

Evidence

Candidate evidence for problem solving could include:

- description of the problem
- tutor records and agreement of standards and approaches
- annotated action plans
- records of discussions
- descriptions of options
- records of reviews.

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