

Core 3 Revision

Functions

- In a function every element of the domain maps exactly to one element in the range.
- In a one-to-one function, every element of the range maps to one in the domain.
- Functions can be expressed in the form $f : x \rightarrow 3x$, which is equivalent to $f(x) = 3x$
- Composite functions:
 $gf(x) = g(f(x))$ and
 $fg(x) = f(g(x))$
- To calculate the inverse of a function, reverse the subject and re-arrange. The inverse will be a reflection in the line $y = x$

Exponentials and Logarithms

- Exponentials of the form $y = a^x$ all pass through (0, 1)
- $y = e^x$ is a special case such that the gradient is equal to the function.
- The inverse of $y = e^x$ is $y = \ln x$
- Growth and decay models are often based around exponentials.

Numerical Methods

- If in an interval $f(x)$ changes sign, then the interval must contain a root of the equation $f(x) = 0$
- $f(x) = 0$ can be solved iteratively by rearranging to form $x = g(x)$ and applying $x_{n+1} = g(x_n)$, where x_0 is close to the root. This method may not converge at the root.

Graphical Transformations

- The modulus operator $| |$ always returns a positive version of the input.
- $y = |f(x)|$ will have no parts below the x axis.
- $y = f(|x|)$ is the graph of $y = f(x)$ for $x \geq 0$, reflected in the y axis.

Trig Functions

- $\sec \theta = \frac{1}{\cos \theta}$
- $\csc \theta = \frac{1}{\sin \theta}$
- $\cot \theta = \frac{1}{\tan \theta}$
- Arc forms the inverse of trig functions, but is limited to ensure a one-to-one mapping is maintained.

Trig Identities

- [See Formula Sheet]

Differentiation

- Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
- Quotient Rule: If $y = \frac{u}{v}$,
$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$
- Product Rule: If $y = uv$,
$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$
- $y = e^x, \frac{dy}{dx} = e^x$
- $y = \ln x, \frac{dy}{dx} = \frac{1}{x}$
- [Trig Rules]